The St. John's Review

Volume XLVI, number three (2002)

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The St. John's Review is published by the Office of the Dean, St. John's College, Annapolis: Christopher B. Nelson, President; Harvey Flaumenhaft, Dean. For those not on the distribution list, subscriptions are \$15.00 for three issues, even though the magazine may sometimes appear semiannually rather than three times a year. Unsolicited essays, stories, poems, and reasoned letters are welcome. Address correspondence to the Review, St. John's College, P.O. Box 2800, Annapolis, MD 21404-2800. Back issues are available, at \$5.00 per issue, from the St. John's College Bookstore.

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ISSN 0277-4720

Desktop Publishing and Printing The St. John's Public Relations Office and the St. John's College Print Shop

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Wholes and Parts in Human Character Joe Sachs

Who are you? What are you? I'm not asking for your name or occupation, but rather for what you're made of, or what you amount to. And I'm not intending to be impolite or impertinent, but to include myself in the question, to turn the attention of all of us upon the philosophic question that touches us most closely. Thomas Aquinas is not hesitant about answering it (Summa Theol. I-II, Q. 1, art.1, resp.): "Man differs from irrational animals in this, that he is master of his actions. Wherefore those actions alone are properly called human of which man is master. Now man is master of his actions through his reason and will." Shakespeare's Hamlet articulates the same thought in praising Horatio, and formulates the contrasting state (Act 3, scene 2, 64-75): "Since my dear soul was mistress of her choice, /And could of men distinguish, her election / Hath sealed thee for herself...Give me that man / That is not passion's slave and I will wear him / In my heart's core, ay, in my heart of heart, / As I do thee." So you amount to either a master or a slave, depending on whether reason or passion has the upper hand in your makeup.

But is this contrast well-founded? David Hume doesn't think so. "Nothing is more usual in philosophy, and even in common life," he writes," than to talk of the combat of passion and reason, to give the preference to reason, and to assert that men are only so far virtuous as they conform themselves to its dictates." But Hume argues that reason has no

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power to oppose passion or produce action, and concludes that "Reason is, and ought only to be the slave of the passions" (*A Treatise of Human Nature*, Bk. 2, Part 3, sect. 3). This deliberate paradox may be more palatable when we note that Hume is only adding force and vivacity to an earlier formulation of Thomas Hobbes, who had written that "the Thoughts are to the Desires, as Scouts and Spies, to range abroad, and find the way to the things Desired" (*Leviathan*, Part 1, Chap. 8). Hobbes and Hume agree that reason is merely instrumental to our primary mode of access to our own good, which can only be irrational.

Now the weakness of reason may be granted by those who would not go along with demoting it to a subordinate role in us. Immanuel Kant reflects that "if…happiness were the real end of nature in the case of a being having reason and will, then nature would have hit upon a very poor arrangement in having the reason of the creature carry out this purpose…And in fact, we find that the more a cultivated reason devotes itself to the aim of enjoying life and happiness, the further does man get away from true contentment" (*Grounding for the Metaphysics of Morals*, Hackett, p. 8; Academy p. 395). Kant sides with reason, but as our guide to becoming worthy of a happiness which can never be realized in the empirical world. We are radically divided beings, in his view, and can never have it both ways.

But an older sort of wisdom is articulated in Plato's *Republic* (esp. 439D-442B), according to which the human soul is not a duality of reason and passion, but has three parts, with the middle part giving it the possibility of wholeness. As described in the *Republic*, this middle part is what is irrationally spirited in us, just as in a spirited horse, but capable of obeying reason, so as to be able to follow its leader like a dog. There is nothing spiritual in this sort of spirit, but there is something that can have dignity, since it appears not only in pep rallies that arouse school spirit, but also as what we call the indomitable human spirit which can rise above any adver-

sity. The republic, that is, the regime or constitution, that Plato's *Republic* is about is the internal human commonwealth in which reason rules and directs the passions by joining with, and giving honor to, our spirited side. Only that third part of us is capable of loving the good and being loyal to it, and the constitution to which it submits is not the despotic one of mastery, but the political rule of persuasion.

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About fifty years ago, C. S. Lewis wrote a book about education called The Abolition of Man. His claim there is that all the ancient human traditions have addressed in some form the middle part of the human being, while the prevailing thinking of the enlightened twentieth century has lost touch with it altogether. If we fear all the attachments of our spiritedness as divisive or sentimental, as obstacles to progress, then we may proclaim our rationality, while in fact we come to be ruled by the lowest common denominators among our appetites, if not by mere caprice. Those progressive educators who fail to understand this may like to call themselves intellectuals, Lewis says (p. 35), but "It is not excess of thought but defect of fertile and generous emotion that marks them out. Their heads are no bigger than the ordinary: it is the atrophy of the chest beneath that makes them seem so." If reason is universal and the passions are generic, then it is the spirited, honor-loving element in our make-up that most of all makes each of us what we are, and a disdain for it, with a consequent neglect of its nurture, leads to the dis-education that Lewis calls the abolition of man.

Now these provisional sketches of the ways reason and passion may stand toward one another in us may serve as background to an exploration of Aristotle's thinking about the same topic. When I came to spend an extended time in close contact with the *Nicomachean Ethics*, I found a number of things that surprised me. After many earlier readings of the book, I would have said confidently that Aristotle believes the healthy human soul to be under the command and control of reason, and that something in us that is neither wholly rational nor wholly irrational makes that possible. As in so many other ways, that is, I thought that Aristotle had followed a path marked out by Plato, making it determinate in his own way. Now I am not so sure of that interpretation. Some distinct and memorable passages in the *Ethics* seem to paint that portrait of what a human being is, but others that are more difficult, more entangled, and more central to Aristotle's inquiry seem to tell a different story. I hope to make both accounts clearer to myself, and to invite all of you to take part in assessing them, both as readings of a text and as reflections of the subject we all have an interest in knowing best.

Now, anyone who has ever taken Plato's Republic seriously, and hasn't been frightened off by its lack of currently popular jargon and currently prevalent opinions, knows that the three-part soul offers a powerful way of analyzing human life. Socrates is challenged in Book 2 to refute the claim that ethical virtue provides a second-best life, a social compromise made among ourselves by the weak, who play it safe to achieve a mediocre and watered-down version of happiness. This argument of Glaucon (358E-359B) is in no way inferior to similar ones made two thousand years later by the social contract theorists, beginning with Thomas Hobbes. But by the end of Book 4, not even halfway through the dialogue, Glaucon admits that his argument has been exposed as ridiculous, and needs no refutation (445A). What has made ethical virtue go from seeming indefensible to seeming unassailable as a thing desirable for its own sake is nothing more than the hypothesis of the three-part soul. With that hypothesis, the examination of the pursuit of happiness shifts from the conflict among human beings to the conflicts within each one of us. That permits Socrates to conclude that virtue consists in "ruling and organizing oneself and becoming a friend to oneself, and harmonizing those things, of which there are three, just like the three notes of a musical chord" (443D).

But Aristotle is not comfortable with partitioning the soul at all. In Book 3, Chapter 9 of *On the Soul*, he says one can

distinguish as many parts of the soul as one wishes, and easily find parts farther apart than the three in the *Republic*; and even those three, he says, cannot be wholly separated, since desire is present in all three of them. In the *Ethics* he is willing to adopt the popular way of speaking of a rational and an irrational part, but even there he cautions that these may be no more distinct than are the convex and concave sides of the same circle (1102a26-32). Aristotle's own investigation of the soul focuses on ways of being-at-work, and on the potencies for them, rather than on parts, but the purposes of an inquiry into ethics do not require that degree of precision.

Why, then, if the division of the soul in any manner is merely imprecise or figurative, does Aristotle prefer to accept a two-part rather than a three-part division? Or, what amounts to the same question, why does spiritedness play so small a role in the Nicomachean Ethics? It is an explicit topic in two places, and briefly in both: in one of them it is treated as an attitude that resembles courage but is not the genuine virtue (1116b23-1117a9), and in the other it is described as a particular kind of lack of restraint, an oversensitivity to insults that is less harmful than a lack of restraint in one's desires (1149a24-b3). Desires are governed by pleasure and pain, while spiritedness is governed by honor and shame. Now over the course of the ten books of the *Ethics*, the topic of pleasure re-emerges more than once; as the inquiry deepens, pleasure itself is seen in new lights, and transforms itself through the growth of human character. By contrast, the topic of shame is set aside early on (Bk. 4, Ch. 9), as something appropriate only to immature and undeveloped states of character. This is one of the clearest signs that Aristotle does not consider ethics to be concerned at all with social pressures, or the imposition of social norms, but to depend upon rising above motives of that sort. He calls it absurd to credit an adult with decent motives if he refrains from certain kinds of acts only out of shame (1128b26-28).

Honor too, the positive opposite of shame, is left behind early in Aristotle's study of ethics, but it is dealt with a little more extensively, and in fact we can observe the very moment when it drops out of the developing account of human character. Very early, in the fifth chapter of the whole work, Aristotle dismisses the honor that is the highest aim of political life as too superficial a thing to be a plausible candidate for the ultimate human good (1095b22-30). Honor shifts with those who give it; one really wants honor from those worthy to give it, and only for things in oneself that are worthy of receiving it, so the true standards that make honor worth pursuing are some sort of virtue or excellence, and the wisdom to recognize it. A little later in the inquiry, though, honor turns up again as entangled with one of the virtues, and the account of that virtue repeats the dialectical motion of Aristotle's earlier analysis of honor, this time as a lived development within a human being. That virtue is a complicated one to understand, just because of its complex relation to honor. Aristotle calls it greatness of soul.

Our times, in which so many people have attempted to deny the existence both of souls and of any form of greatness, offer us no clear equivalent of Aristotle's phrase. The old Oxford translation called it pride, which might capture the greatness of soul Homer portrays in Achilles; some translations have called it high-mindedness, which might capture, or rather caricature, the greatness of soul Plato makes visible in Socrates. The worst translation of the phrase, magnanimity, simply plugs in Latin equivalents of each of its parts, and gets a result that has a totally different meaning in English, in which generosity is the primary element. Brother Robert Smith once suggested Charles de Gaulle as the twentieth century's pre-eminent great-souled man, and generosity was no part of the reason. Outside the Ethics, Aristotle mentions both Achilles and Socrates as great-souled, the former because he tolerates no insults, the latter because he is above caring about good or bad fortune (Posterior Analytics 97b1426). These may seem too different to be one virtue, but Aristotle sees what they share as a correct sense of one's own worth, when this is in fact great. It is not a mere feeling of self-esteem, but something that has to be earned. But Achilles appears to have earned it by deeds on the battlefield that subdued other people, while Socrates earned it through talking and thinking that conquered nothing outside himself. The former demands honor as a thing more precious than life, while the latter disdains to claim any honor at all, and keeps insisting that his only distinction is knowing that he doesn't know anything.

These examples are no more puzzling than Aristotle's account of the virtue. He says first that the great-souled man is concerned with great things (1123a34), and then quickly decides that those great things are great honors (1123 b20-21), but it takes him only a little bit of argument to conclude that for such a person, if he is genuinely worthy of what he claims for himself, "even honor is a small thing" (1124a19). Putting together the examples with the argument: (a) the great-souled man is obviously a lover of honor like Achilles, and (b) by being great-souled through and through, Achilles must become a disdainer of honor like Socrates. The argument that runs from 1123b26 to 1124a19 is one of the pivotal passages of the Nicomachean Ethics, the place in which honor stakes its maximum claim, and earns it by rising above itself. Greatness of soul is puzzling to us not because it belongs to an obsolete aristocratic culture but because it is inconsistent in itself. Aristotle takes it seriously not by way of stooping to an audience of Athenian gentlemen but because it is something serious for any serious human being at any time or place. It is one effective first stage toward the full development of human character. We might make this clearer by considering two of the pre-philosophical thoughts about virtue that form part of the background of Aristotle's inquiry into ethics.

One of those common opinions is a widespread acceptance that four virtues are the cardinal ones, namely wisdom, courage, temperance, and justice. In the Nicomachean Ethics, Aristotle gives justice and wisdom each a book to itself, and courage and temperance together about half a book. By this crude test of length of discussion, greatness of soul gains the fifth place, as either the fifth cardinal virtue, or the ambiguous virtue that almost, but not quite, belongs among the primary elements of human character. Another piece of popular lore that Aristotle alludes to (1095b17-19) may help us understand what gives greatness of soul both a high rank and a secondary position; it is an old Pythagorean parable of the three kinds of life. The parable likens the ways of life to the three kinds of people who go to the Olympic games. The greatest number go to buy and sell things; a smaller number go to compete. The third group, and by this account the smallest number, go simply to watch. Aristotle calls the corresponding lives those devoted to enjoyment, to politics, and to contemplative thought. Their aims are bodily pleasure, external honors, and knowing.

Now it is clear that these three lives stand behind the Republic, in which Plato's Socrates seeks to blend their three aims into one soul, understood as an interior polity or commonwealth. But Aristotle's approach seems different. Even the bodily desires are treated in the *Ethics* as capable of being educated and redirected, to gain a deeper satisfaction than they find in their crudest form. This is not a matter of moderating them, of making compromises with them, or of reason's holding them in check by means of its spirited ally; according to Aristotle, the temperate person has no harmful desires (1119a11-15; 1146a11-12). Similarly, I think, the person whom Aristotle considers worthy to be called greatsouled has no craving for honor. He has, instead, internalized his standard of worthiness. The brief argument I referred to a moment ago, in which honor must abandon its claims, is Aristotle's way of showing the honor-loving person that his true satisfaction lies nowhere but in well-grounded selfrespect. I believe that Aristotle has here discovered what has come to be called a sense of honor, a meaning that the Greek word for honor never had. Aristotle does not dwell on this discovery, but pushes it one more step, into the realization that even an internal tribunal of honor is an inappropriate standard by comparison with the virtues to which it looks.

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One of the most interesting things about Aristotle's treatment of greatness of soul is that it generates a new series of lives that replaces the three in the Pythagorean parable. It is the first place in the *Ethics* in which Aristotle finds the virtue he is discussing to be a center around which all the virtues must be arrayed. Greatness of soul is not so much one virtue as one way toward the wholeness of character. Aristotle asks, for each of the virtues one-by-one, whether it would not be completely ridiculous for anyone who lacked it to have any claim to greatness of soul (1123b33-34). To someone wrapped up in the craving for honor, Aristotle asks, in effect, "Do you just want to have it or do you want to deserve it?" It is strange but true that anyone who really wants honor is bound to say that he doesn't just want honor, since any other answer would lose him what honor he had. Imagine a politician saying, "Vote for me because I love winning elections." A serious person, who seriously craves honor, cannot help concluding that honor is not the most serious thing.

Now I'm not suggesting that Aristotle thinks that all one needs to do to transform someone consumed with a drive to be honored is to ask him a couple of questions. As I mentioned before, the questions and their answers seem to reflect a lived process of discovery and development. There is an exact analogy in the life of enjoyment. When Aristotle has made his first provisional definition of happiness as activity in accordance with virtue (1098a16-17), he observes that this has the properties sought by the life of enjoyment more fully and genuinely than does the indulgence of bodily pleasures (1099a7-21). The difference is that those who pursue the crudest enjoyments are always chasing after pleasure as a thing external to themselves, while those who live the virtues have pleasures that are internal and durable and always present. Aristotle calls the latter the pleasures that are natural, because they are pleasant in themselves, just by being themselves. He later analyzes the cruder pleasures of eating, drinking, and sex as attempts to magnify the natural pleasure inherent in good bodily condition by violent departures from it and restorations of it (Bk. 7, Ch. 14). This sort of thing, which bar owners encourage by offering free salty snacks to make people drink more beer, can't really fool any grownup for long, and can only continue to appeal, as Aristotle says, to those who have found no other sources of pleasure (1154b5-6). Such people are not worthy challengers to the title of happiness, entitled to the pretentious name of hedonists; they are merely failures as pleasure seekers, failures by no standard but their own.

The analogy in the case of greatness of soul is evident in the contrast between chasing after externally bestowed honor and settling into a life that is inherently honorable. It is not such a stretch to say that Achilles can achieve his aim only by becoming more like Socrates. The 19th-century philosopher Friedrich Nietzsche (Beyond Good and Evil, §212) believed that Socrates had pulled off a brilliant piece of one-upmanship, making himself appear superior to the aristocrats who thought they had a monopoly on superiority. But in Aristotle's view this was no transformation of values, but a natural dialectic that leads the honor seeker beyond the random honors that can be won to the life of natural and selfsustained honor that accompanies the virtues from within. This explains why Aristotle leads the discussion of greatness of soul to the need for a life that demands the whole of virtue. The honor seeker goes in search of an isolated prize, and if he perseveres, and keeps his eyes open, he finds instead a life. The next prize or the next victory accomplishes nothing more durable than would the next beer. As the latter might be the perpetually renewed object of the deluded pleasure seeker, the former perpetually enslaves the competitive athlete or politician who has not learned that he can honor himself.

So, finally, we can see why Aristotle doesn't build good character on the spirited part of the soul. Good character is achieved only when spiritedness subsides, and is built not by harnessing spriritedness but by overcoming it, in letting it find its truer and deeper satisfaction. Greatness of soul is a way of life not in the sense of being, for some people, the primary aim to which all virtues are subordinated, but in the sense that it finds its end in making those virtues themselves primary. Aristotle gives greatness of soul not a static portrait, but a dynamic impulse toward its aim. But how does he do this? The appeal he makes is, oddly, neither to reason nor to spiritedness. There would be nothing unreasonable, and certainly nothing self-contradictory, about saying one wants honors more than one wants to be worthy of them, and there must be something deflating to an aroused spiritedness in accepting a giving up of competition. Defeated politicians and retired athletes talk as though being unable to keep winning things is like death to them. What Aristotle is addressing is a whole human being who is driven to achieve great honor; his question brings such a person face-to-face with his own judgment of what makes one honor greater than another.

Greatness of soul is the first and lowest portrait in the *Nicomachean Ethics* of a complete way of life that replaces the untenable claims of the life of bodily enjoyment. It is not the last or the only portrait of such a life. The craving for honor is not a necessary precondition of the development of good character, but only one human road that gets there in the end, for anyone who is serious enough about achieving satisfaction in life. Aristotle's tactic in leading the honor-lover to virtue is no different at bottom from his means of exploding the claims of bodily enjoyment, and it is evident in the first sentence of Book 1: "It has been beautifully said that the good is that at which all things aim." His tactic is not a nego-

tiation for turf among separate parts of a soul, each of which wants its separate good, but a constant recurring to the question, asked of the whole human being, What end is being sought in each good object? What makes one pleasure more satisfying than another? What makes one honor greater than another? Whatever in us responded to these visible goods by pursuing them remains sovereign in judging them. The task is to free it from enslavement to the familiar and habitual, and to lift its gaze to the widest view of its possible choices.

Now if this conclusion sounds plausible to you, that's remarkable. It implies, first of all, that Aristotle believes that the well-ordered human soul is ruled by desires and not by reason, and second, that he regards habits not as the source of good character but as obstructions to it. These are not standard readings of the *Nicomachean Ethics*, and that's putting it mildly. But we've arrived at them honestly and we're stuck with them; we have no honorable course other than to follow where they lead us.

I mentioned some time ago that there are some passages in which Aristotle seems to say that reason needs to rule us. They begin very early, with his first conclusion that the minimal condition of a satisfying human life is to put to work that in us which has reason and listens to reason (1098a3-5); and they continue very late, into his final description of the happiest life, with assertions that the intellect is the best part of us, is the part that naturally rules and leads us, and is even what a human being is most of all (1177a13-15; 1178a6-7). But in between these passages, one of the things Aristotle has done is spell out what he means by intellect, and it is not a synonym for reason. Moreover, in discussing a number of the virtues, he has stated what power in us has the ultimate say in judging what is right, and it is not reason. And most conclusively, although almost everything else he says in the Ethics is dependent on the dialectical process of inquiry that leads to it, he has along the way formulated unequivocally and categorically what constitutes a human being as a source of action, and again it is not reason.

The most straightforward way one might imagine that reason should rule us is to think each of our actions should be deduced from some principle. Aristotle does talk this way at one point, but the only complete example he gives belongs to a vice rather than to a virtue: one ought to taste every sweet thing; this thing in front of me is sweet; and one more act of gluttony takes place with apodeictic necessity (1147a29-31). But one point that Aristotle makes here is that even though the major premise is a universal proposition and the conclusion is an action, the minor premise is something particular, and all such things are governed by sense-perception (1147a26). In two places Aristotle says that there can be no rules for right action (1113a31-33; 1137b29-32), and in two other places he says that the judgment or decision that determines all matters of action is in the perceiving (1109b23; 1126b3-4).

But what sort of perceiving is this? My dog might have better eyesight than I do without being able to see what's best to do in any situation. While the dog might associate past experiences with present perceptions, the human power of perception is infused with an intelligence and an imagination that lets us grasp particular things as instances of universals. Aristotle's word for this (epagoge) is nearly always mistranslated as induction, which suggests imposing some general formulation on the thing in front of me, but he means something more direct, by which I see the thing in the first place as a thing and an example of a kind. The power in us that operates in and through our perceiving to behold the universal directly is called *nous*, or intellect. The intellect thus supplies the starting points of all universal reasoning (Bk. 6, Ch. 6), which are at the opposite extreme from the particulars (1142a23-27), but at the same time grasps those very particulars (1143a35-b5). Practical reasoning about action, like universal reasoning about the way things are, cannot begin

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unless and until something in us that is not reason provides starting points to reason about. In both cases this is the intellect, the power that beholds the ultimate invariable thinkable things in contemplation but also the ultimate changeable particulars in perceiving. Jacob Klein once compared thinking to walking: taking steps is like reasoning, but having something to step off from is like intellect. Reason is ruled by intellect.

Now it might sound as though I'm quibbling. When you hear the claim that reason is the proper guide and ruler of our lives, you probably understand reason to mean the whole power by which we dispassionately recognize what is evident, whether directly or through any number of steps. The important thing is that we should be led by something that is impartial or, we may wish to say, objective. But that isn't good enough. What's needed is to see things in relation to us and our ends. Aristotle says that for knowledge to accomplish its work well, it has to guide it by discerning the mean in relation to us (1106b5-9), which is a mean in the sense that it does not overshoot or fall short of our natural ends. That's what there are no rules for. That's what has to be perceived. That's the practical work of intellect.

One of the most persistent themes of the Nicomachean Ethics is that achieving any aim in life depends upon seeing straight, seeing things the way they are. But at least five times (1099a22-24; 1113a25-33; 1166a12-13; 1170a14-16; 1176b24-26), Aristotle tells us that it is only someone of good character who is capable of seeing straight. The way most people see things, he says, is distorted by the false appearance of things that promise pleasure (1113a33-b1), and he notes that overly spirited people are too quick to perceive things as insulting when they aren't (1149a25-34). It is obvious that fear has the same effect, since he says the coward is afraid of everything (1116a2-3). In fact, one way to see what Aristotle means by virtue of character is to imagine all the ways we might be so mastered by some kind of feeling that we have no capacity to see things for what they are. The

virtues of character are the stable conditions that free us, not from those feelings, but from being unable to make choices about them. A courageous person can distinguish what is worth being afraid of, and still has the option to act for an end beyond it. A temperate person can eat, drink, and be merry when *he* chooses to and not every time an opportunity for indulgence is present. A great-souled person can avenge an insult when his good sense and courage lead him to, and not as a reflex reaction. Some translators call these states of character moral virtues, but that misses the point. They are practical virtues; without them action is impossible.

Intellect is not stronger or sharper in a person of good character, but only in such a person is it free to operate. This is a negative sort of dependence. Good character clears away the bad habits that obstruct the sight of the intellect. But there is also a positive dependence. A person of good character is not someone who is neutral about pleasant or frightening or insulting things-thinking itself moves nothing, Aristotle says (1139a35-36). Good character combines intellect that is clear-sighted with desires and aversions that are good, in the sense that they are rightly proportioned to the end of one's own happiness. Free choice of action depends not on objectivity or independence from our inclinations but on the presence and participation of all our desires. Aristotle says that for a choice to be good, one's desire must be right (1139a30-31), but the rightness of desire is entirely analogous to the clarity of intellect. Right desire is unobstructed desire.

Aristotle does not understand our desires to be a disorderly mob that needs to be ruled, but the natural components of a life in which they all must have full scope to act. Once we have taken responsibility for our own lives, we can develop what Aristotle calls active conditions of the soul, and it is these active conditions that make up character. Formation of an active condition (*hexis*) begins with isolated choices (1103b14-23), such as refraining from a harmful pleasure or enduring a frightening situation. Sticking by these choices requires effort, because they go against the grain of the masses of bad habits that have already become ingrained in all of us before we have any power of choice. All deliberate and positive habituation, from parental training, the laws of the community, or the effort of self-discipline, is for the sake of cancelling out the blind and passive earliest habituation that no one intends, that comes just from the reflexive slackening of the tension of uncomfortable feelings. The active condition is fully formed when no more effort of self-restraint is needed. Knowing how we want to be in life and action is not enough (1105b2-3), but being that way is still up to us (1114b21-23). Each of the virtues of character is an active condition in relation to some of our feelings and impulses; Aristotle's claim is that for each of these active conditions there is an unimpeded way of being-at-work, and that happiness is the being-at-work of them all (1153b9-11). When cleared, by habit, from the distortions of bad habits, our habit-free desires constitute ourselves as we are by nature; if these natural desires are thwarted our lives are stunted.

Aristotle's picture of the healthy human soul is not a three-part hierarchy but an equal partnership of everything in us. The center of the Nicomachean Ethics, early in Bk. 6 (1139 b4-5), is Aristotle's unequivocal definition of a human being as the source of action in the act of choice, that can be equally well described as intellect fused with desire or desire fused with intellect (orektikos nous or orexis dianoetike). Neither side can have the upper hand. The dictates of reason as master are as misguided as the caprices or addictions of an overmastering passion. Even a state of compromise, in which our rational and irrational sides each give up a little of what they want to get a little, misses the mark. What Aristotle is talking about is a genuine whole, in which the parts are not externally connected but internally infused with one another. Thinking and deliberating must come to be present within our desires, if they are to be directed toward their true ends, rather than toward immediate delusions. But it is equally true

that intellect would be useless if it were objective or neutral, and it has to be led by desire to see things as ends, in relation to ourselves. The formation of the component parts of character is not a process of reasoning, not a discipline of the passions, and not a combination of the two; it is a gradual and mutual development of thinking and desire as enlightened by one another, each led by its partnership with the other to its own end.

Now if this non-hierarchical picture of the soul strikes you as un-Aristotelian, your uneasiness is understandable and deserves a response. Aristotle seems always to be talking about rankings of things as higher and lower, and we saw earlier that he refers to the intellect as the best part of ourselves. And in the Politics, he says that whatever is composed of a number of things and becomes one has a ruling part and a ruled part (1254a28-32), and he gives the very example we are exploring; he says explicitly that intellect rules desire (1254 b5-7). But he is careful to say that this is not the rule of a master or a monarch, but political rule. He explains soon afterward that by political rule he means rule over equals (1255b20), and he later adds that political rule requires an alternation or sharing of ruling and being ruled (1277b7-16). This is a working-out of his understanding that the political community is not merely an alliance for the sake of promoting exchange and preventing injury, but is a genuine whole. All the more so, one might suppose, is the human soul not just a bundle of capacities and desires, but a unity.

It is in Bk. 7, Ch. 17, of the *Metaphysics* that Aristotle distinguishes anything that is truly whole and truly one from a mere heap (1041b11-12). His example of a genuine whole is a syllable. To see what he means, take the first syllable of the word metaphysics, and try to sound it out by parts. You will hum, and then make an exclamation that might sound either dismissive or interrogative, and then make an explosion of breath. No matter how fast you make these noises in sequence, they will not make the simple syllable met-. It is not a sum of parts but can only come into being as a whole. You cannot pronounce it at all unless you make an m sound that is already shaped by the following e, and sound the other letters similarly, not as isolated bits of noise but with the whole syllable present in each of them. When you try to remove the part from the whole it becomes something different; it is not at all like plucking one marble off the heap. Something is whole most of all, Aristotle says, when its constituent parts are distinct only potentially (1023b32-34), and in the *Politics* he gives the famous example that a human being removed from political community is not human at all, but either a beast or a god (1253a26-29).

It follows that dispassionate reason is not the human thinking power at all, and unintelligent desires are not human desires. Remember that Aristotle cautioned us, when we first separated a rational and an irrational part of ourselves, that we might be trying to separate the convex from the concave side of the same curve. We saw earlier how the positing of external honor as the greatest of goods led back into the soul, and into Aristotle's first depiction of the virtues of character as composing one whole life. There are four more occasions in the Ethics in which Aristotle leads some partiality in the soul back to a wholeness of character. The second type of person who needs to be so led is also focused outward, but on the good of other people (1130a3-5). Aristotle does not have a word of criticism for such a person, and even calls him the best human being. He merely points out that his aim requires the presence of all the virtues, and goes back to discussing those virtues. But when the examination of all the virtues is said and done, it is not this life that is called best, but an entirely different one. Like the assertion that honor is the greatest of external goods, that is made in Bk. 4 (1123b20-21) and quietly replaced in Bk. 9 (1169b8-10), the claim that the person devoted to doing justice to others is the best human being is also refuted just by being unable to withstand five books of further inquiry.

I find it enormously instructive that Aristotle permits justice to stake the same claim to being the highest good that honor does, and makes it fail. It is not in the long and detailed discussion of justice as one particular virtue among many that this occurs, but in a brief preface on justice as a way of life. Aristotle mentions one celebrated ancient political leader (Bias) in this connection, but a clearer example for us might be a Roman, Cato the Younger. Plutarch says of him (Lives, Modern Library, p. 920) that he was inspired and possessed by a devotion to every virtue, but especially to that "steady and inflexible justice that is not to be wrought upon by favor or compassion." He exemplifies the immersion in political life that is governed not by honor but by duty. I mention this only because it helps one see that Aristotle's ethics of character has nothing to do with impersonal duty. Aristotle is constantly speaking of what one ought to do, but for him this imperative always arises only from within, from the need to fulfill and put to work all our own powers to achieve happiness. I noted earlier that the Aristotelian virues of character are not moral virtues but practical virtues. I am suggesting that morality is a misunderstanding of Aristotle's characterbased ethics. Morality, in the sense of doing right by others. follows from the practice of the virtues in the same way pleasure does, but doesn't work as its end. Why should you not be a thief? Because that is your duty to other people, or because to be one would distort your own life and make you fail to gain your own happiness? Or put it the other way around. Would you rather have a neighbor who grimly and dutifully refrains from your property even though he might covet it, or one who has found a life in which your property is of no interest to him?

The third life Aristotle describes is that of practical judgment (1144b30-1145a6). This is usually translated as prudence or practical wisdom. It is the virtue of intellect that is a precondition of every virtue of character, but it in turn cannot come into being without the presence of all the virtues of character, which permit it to discern its ends. This mutual dependence is the clearest evidence that the intellect is not uppermost in practical life, but there are certainly people who seek not to rule others nor to serve others, but to be the guiding intelligence behind political life. William O'Grady used to say that whenever he read Machiavelli's Prince, he always wondered whether that book was more like housebuilding or flute-playing; that question is Aristotle's way of distinguishing activities with external ends from those whose ends are in themselves. Themistocles, an Athenian general described by both Herodotus and Thucydides, was a great conniver who seemed happiest when he had successfully manipulated everyone on every side of a conflict; he found political maneuvering an end in itself. But Aristotle says at the end of the Ethics that there has been no one up to his time who had all the requirements of the political art (1181b12-13). As with honor and duty, his purpose seems to be to show those who aim at superior practical know-how as their highest end that the only role of the practical intellect in a successful life is a more modest one, on an equal footing with all our desires.

Now the progressive overcoming of the claims of greatness of soul, justice, and practical judgment to be the pre-eminent virtue seems to exhaust all the motives for which one might call the political life the highest life. Aristotle does not collect them in this way, but he concludes in general that political life is unleisured and always aims in part at a happiness that is beyond itself (1177b12-15). He contrasts it to a contemplative life, which he argues at length is the best and happiest human life (Bk. 10, Chs. 7-8). This produces the greatest controversies among readers of the Ethics, since it seems so odd that any philosopher would devote 98% of a long and lovingly worked-out inquiry to an examination of the practical life, only to turn his back on it at the culmination of the work. Without entering into any such controversy here, I think we are in a position to see the general lines of a solution. There is no contemplative life for anyone who has not achieved a good character, not only because his desires would get in his way if they were not set right, but also because human nature itself is one of the unvarying things that the contemplative intellect needs to understand; in that odd way, even in its non-practical employment the intellect needs a respect for and partnership with our irrational desires, since they are among its teachers. A human being is intellect and desire, inseparably intertwined.

But also, I have so far skipped the fourth of Aristotle's five depictions of the whole life of virtue. Between the three versions of political lives and the contemplative life there is an ambiguous middle life. The life of friendship in the full sense also appears to involve the practice of all the virtues of character (1157a18-19, 29-31). But Aristotle does not say that friendship requires the prior presence of those virtues, but that friendship itself, in its proper sense, is present to the extent that the friends possess the virtues (1156b8-9; 1157a30-31). He seems to mean that the friendship deepens as the characters of the friends do. As with greatness of soul, the picture seems dynamic rather than static, but not because friendship is an inadequate motive. Like contemplative wisdom, friendship seems to be a final and all-inclusive end for the whole of life; and Aristotle says it would be absurd to imagine the happiest life as one devoid of friends who are loved for their own sake (1169b16-28; 1156b9-11). Friendship directly supersedes greatness of soul, since Aristotle concludes that it is not honor but friends that are the greatest of external goods (1123b20-21; 1159a25-26; 1169b8-10). But it is also precisely friendship that supersedes justice as the highest aim and bond of political life, since Aristotle says that where there is justice there is still a need for friendship, but where there is friendship, justice is not necessary (1155a22-28). Even the life devoted to the exercise of the practical intellect seems to find its highest fulfillment in friendship, since Aristotle says that one can contemplate his own active life best in the actions of his friends (1169b30-

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1170a4). Everything a human being seeks in political life is found only in friendship.

The relation of friendship to the contemplative life might seem to be that of second best to best, and Aristotle says just that (1178a5-9), but this is not a case in which the lower is left behind in the achievement of the higher. As we have noted, the claims of friendship within happiness are themselves final and indispensable, and a contemplative human being does not cease to be a human being. And in a certain sense, friendship has a claim to be not only of equal rank with the pure life of intellect, but even superior to it. Aristotle argues near the end of Bk. 9 that the highest life, spent in the being-at-work of our highest powers, is deficient if it is not expanded through being shared with friends (1170b7-19). The motion throughout the Nicomachean Ethics is toward the greatest wholeness of the life of everything in us, and this can be achieved neither by a friendless life of contemplation nor by any practical life shared between friends that excludes the enjoyment of knowing. Aristotle says early on that the truth deserves higher honor than do one's friends, but he does not say that either of them ever cease to be loved for their own sake (1096a16-17); in one of the most powerful indications of the destination of the whole work, he speaks of the truth and his friends together in the dual number, a resource the ancient Greek language had for naming things that are more than one but inseparable.

So should reason, or our whole thinking power, rule the soul? Clearly Aristotle doesn't think so. Thinking guides desire, but desire guides intellect, and intellect guides thinking. Under the guidance of thinking, desire can find itself directed beyond bodily pleasures and political honors to an end in friendship. Under the guidance of desire, intellect can see the best choices before it in all their particularity. And we are, first and foremost, choosing beings—not rational animals, not political animals, though we are those things too. Choice cannot be full and unimpeded if anything in us is held back or held down or has not become fully aware of its nature. Ultimately, for us, even contemplation is a choice, in which the activity chosen and the whole desiring being that chooses it are equally necessary and equally sovereign.

Addition and Subtraction without End in Oresme's *Quaestiones super Geometriam Euclidis* George Anastaplo

This essay is an explication of the first two questions of Nicole Oresme's *Quaestiones super Geometriam Euclidis*, a work the Parisian master is thought to have composed around 1350 in conjunction with his teaching in the Faculty of Arts.¹ In these questions, offered as commentary on Campanus's edition of the *Elements* of Euclid, Oresme presented a formal understanding of infinite diminution and augmentation of magnitude that far exceeded what is to be found in the writing of either Campanus or Euclid.²

In what follows, I first consider Oresme's approach to infinite diminution, its context and mechanics. I next examine his presentation of infinite augmentation and reconstruct the understanding in which it was founded. As a conclusion, I offer some reflections about the form of Oresme's work and speculate about the implication it might have for his notion of mathematics.

Oresme began his *Quaestiones* in a peculiar way:

Concerning the book of Euclid it is first asked about the dictum of Campanus in which he laid down that magnitude decreases without limit.³ It is sought first whether magnitude decreases without limit according to proportional parts.⁴

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To a modern reader, this initial question is unexpected. On the surface, it would seem more appropriate to begin a work on the geometry of Euclid by addressing the geometer himself instead of his commentator. In its context, however, Oresme's question makes a credible starting point.

The propositions of the first Book of Euclid's *Elements* are prefaced by definitions, postulates and common notions. In Campanus's edition the last common notion and the first proposition are separated by the following editor's note:

Moreover it must be recognized that beyond these common conceptions of the mind, or common opinions, Euclid has passed over many others which are incomprehensible in number, of which this is one:

If two equal quantities are compared to some third of the same kind they will both be at the same time alike greater than that third, or alike less, or both equal to it.

Another is this. As much as some quantity is to some other of the same kind, so much [can] some third be to some fourth of the same kind.

This is universally true in continuous quantities. Either the antecedents will be greater than the consequents or less. For magnitude decreases without limit; in numbers this is not so. But if the first is the submultiple of the second, there will be some third alike the submultiple of some fourth, since number increases without bound just as magnitude decreases without bound.⁵

Of his two additional common notions, Campanus was content simply to assert the veracity of only the first. The second he sought to make credible. In a ratio, the antecedent term will be either greater than, equal to, or less than the consequent term. Campanus apparently thought that in the case of equal antecedents and consequents, the existence of a fourth term, bearing to some third term the relation determined by an original pair, should be obvious. With regard to the cases of unequal antecedents and consequents, however, he was less sure. For these he introduced comparisons calculated to quiet whatever doubts might arise. Thus, the possibility of unlimited augmentation in number was called upon to persuade one of the existence of a fourth term when the consequent was superior to the antecedent, just as the possibility of unlimited diminution in magnitude was offered as an aid to the acceptance of its existence in the inverse case.

The possibility of unlimited augmentation in number is clear to anyone with the ability to count. The possibility of unlimited diminution in magnitude, in contrast, is less obvious. On first consideration, it is unclear how Campanus might have envisaged a magnitude to decrease without limit or what he intended to be removed from it to make it shrink so small. Indeed, it is not immediately apparent that this kind of decrease is possible at all.

Oresme's first question finds its basis in this quandary. To be sure, Oresme focused the doubt into a specific query worded in carefully chosen terms. Nevertheless, from the response that is made to the question, the concern just sketched is clearly the one Oresme intended to address.

In this and other examples of treatises written in the *sic et non* style of the disputation, the author's teaching is cast in the form of a dialogue between a student and his master. Typically, the student is assumed to possess both a ready capacity for argument and the youthful courage to speak authoritatively in words not fully understood about matters only partly appreciated. Here both of the virtues of the student are manifest:

There are not an unlimited number of parts in a continuous [magnitude], therefore neither is there

an unlimited number of parts of the same ratio. The antecedent is obvious, since otherwise [the magnitude] would be without limit. The consequent is obvious, since any proportional part is a part of the same quantity as any other [proportional part of that magnitude], therefore the parts of the same [magnitude] are of the same ratio and of the same quantity. (1. 9-14)

Apparently unaware of what proportional parts are, but certain of the impossibility of the limitless subtraction of one finite quantity from another, the student confidently asserts that a proportional part is a constitutive part, that is, a part which, with an integral number of equal counterparts, makes up the whole. Interpreting the words "ratio" and "proportional" as referring to the number of parts into which the whole is divided and thus as denoting the size of each part with respect to the whole (and not with respect to some other part), the objector puts his conviction to work and argues that since in the same whole parts in the same ratio are always equal to one another, the question should be answered in the negative.

Just as the student in this genre tends to have a prescribed attitude and level of ability, the master has a predetermined role as well. He is cast as the guardian of a comprehensive understanding who is responsible for the introduction of his charge into the aggregate possession of scholarship. Because of his learning, the master's arguments never have the singlemindedness of those the student is apt to make. Nor are they the same in focus. Where the student, because he is a schoolboy, is concerned to make a judgment based on his immediate perception and assessment of things, the master, because he is a pedagogue, aims at conveying an entire approach to conceiving the matter on which the question touches and then, only derivatively, turns to the question itself.

The reply the master makes here divides into three parts. In the first he cites the authority of Campanus. Looked at in one way, all the master's efforts are directed toward showing why a man called "*perspicacissimus*" might have said "magnitude decreases without limit." Thus, in the second part he works to present the background considerations in terms of which Campanus's dictum must be seen and then demonstrates its truth. Having reconciled infinite diminution with reason, the master finally returns to the student's objection and inspects it in terms of the understanding he has just established. In what follows, only the major features of the master's exposition have been sketched. Discussion of corollary conclusions, replies to objections and the like has been omitted in the interest of brevity.

Two groups of statements provide the background to the demonstration of Campanus's dictum. One enumerates things "that must be noted," the other those that "must be supposed." First among the former is that:

Parts are called proportional with respect to a continuous proportion; and that such a proportion is a likeness of ratio, as is said in the commentary to the ninth definition of the fifth [book] where it is said that a [proportion] is held between at least two ratios; and for this reason Euclid said that the minimum number of terms in which [a proportion] is found is three and the maximum number cannot be given, since it goes on without limit. (9. 20-26)

Only Definition nine of Book five is cited in this passage, nothing else. What must be noted first is not the definition of "proportional part," "ratio" or "likeness of ratio," but simply the fact that proportional parts, whatever they might turn out to be, have to be understood in terms of the relation which binds them together with other parts of the same proportion. Thus the note continues:

From this it follows that properly, one does not speak of a proportional part, or of two propor-

tional parts, but there must be at least three [proportional parts] and there could be an infinite number. (1. 26-28)

The error the student made was to confuse proportional and constitutive parts. What this means is that he mistook the kind of part which is known by the relation it bears to other parts for one which is known by the relation it bears to the whole. Once the character of the student's error is recognized, the intent of the statement just presented is clear. Its goal is to spur the replacement of one notion of part with another, and it works to do so by focusing attention on what is distinctive about this second kind of part.⁶

Since every constitutive part is an integral portion of the whole of which it is a part, the presence of one implies the presence of the others. For such parts division is singular, and once it is accomplished for one it is, in effect, accomplished for all. With the acceptance of the alternative notion of part as proportional part, however, it becomes apparent that division will have to be conceived differently. Just how, the second and third notes lay down:

Second, it is answered that division according to such proportional parts occurs in as many ways as there are continuously proportional [parts] and there are as many of these as there are ratios, which are without limit....Third, it is said that a line and any continuum can be divided up into such parts. A line can be divided up in two ways because it has two ends and such parts can start from either one. A surface can be divided up in an unlimited number of ways and the same [is true] of a body. (2. 1-10)

Seated in some quantity as the result of division, proportional parts may establish the relation of any ratio. And how does the seating division take place? It takes place successively; part by part, it radiates outward from one extremity of the quantity to be divided toward the opposite one. Unlike the immediate and total division that yields constitutive parts, this division is a process, not an event.

Because of these things to be noted, the consideration of the question has been shifted into the realm of continuously proportional parts and successive division. The statements of things to be supposed progress further into this realm and posit peculiarities of its determination later used in the proof of Campanus's comment. Rather than work through each of them in isolation, however, these statements will be viewed in terms of their function in the demonstration.

This is the assertion to be proved:

The first conclusion is that, if some part is removed from a quantity and from the remainder the same part is removed and from the second remainder the same part is again [removed] and so on without limit, the quantity through this manner of endless subtraction will be exactly consumed, neither more nor less. (2. 23-26)

It is important to be clear about the situation just described. Let a be the total quantity. In the first division, some part of a is removed to leave b. In the second, the same part of b is removed to yield c that was formerly removed from a to yield b, and so on for d, e, and the rest. The results of such division can be schematized in this way:

a = whole b = first remainder = a - a (part removed) c = second remainder = b - b (part removed) d = third remainder = c - c (part removed) (where "part removed" is conceived as a fractional multiplier)

What is to be proved is that the *excesses* taken away at each stage to leave the remainders will eventually exhaust the quantity.

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The proof itself runs as follows:

The whole assumed at the start and the first remainder and the second and the third and the rest are continually proportional (as can be proved by arguing from a transformed ratio). Therefore there is in this case, some ratio and as much again and so on without limit.

Thus it follows from the second supposition that the ratio of the whole to the remainder increases without limit since it is composed with itself.

And one term, that is the whole, is imagined not to change; therefore, according to the first supposition, the remainder is diminished without limit.

And therefore the whole of the quantity is precisely consumed. (2. 26-34)

In the divided magnitude the relation of the whole to the first remainder is reiterated in the relations of each remainder to the one that follows it. In terms of the quantities *a*, *b*, *c*, *d* and so on, this conclusion can be written as:

a:b=b:c=c:d...

The second supposition asserts that:

If to any ratio another [equal to it] is added and then another and so on without limit, that ratio will be augmented without limit and this applies to all quantities.⁷ (2. 17-19) In the divided magnitude, the ratio of the whole to each remainder is the sum of the whole to the first remainder taken as many times as division must be made to produce that remainder. For example,

since a : b = b : c = c : d (a : b) + (a : b) + (a : b) = a : d $a^3 : b^3 = a : d$ (1) (2)

The consideration of this kind of expression provides the basis for the crucial inference in the demonstration of Campanus's dictum. In such expressions, a, the whole quantity, plays two roles: it is both the basis of the first term of one ratio (ratio 1) and the first term simply of the other ratio (ratio 2). Now the calculation of ratio 1 through a continued addition of a ratio with itself is exactly the calculation which provides the situation of the second supposition, and so what is posited there can be inferred here, namely, that the series of ratios $a: b, a^2: b^2, a^3: b^3...$ displays a limitless augmentation of ratio. But, consider the role of the ratios of this series in expressions like the one above. There they are used to "measure" the quantity of ratios like ratio 2, which express the relation of the whole magnitude to some remainder. Because of this, it can now be inferred of the series of which ratio 2 is a member, that the succession of ratios in it too displays limitless augmentation.

Let us stop at this point and take stock of the argument. So far, a process of division has been established in a quantity and the tendency of ratios of the whole to a succession of remainders has been described. Now it is left to describe the tendency that the remainders themselves reveal as division is continued indefinitely. And to do this, the first supposition is employed. It is a bridge between considerations of relation and those of quantity:

And if some ratio be augmented without limit, the first term remaining unchanged, the second will be

diminished without limit. This is clear since the ratio can be increased without limit in two ways, either through the limitless augmentation of the first term or the limitless diminution of the second term. (2. 11-16)

Of the two cases, it is clear which obtains here. In the series of ratios established through the division, the whole is the unchanged first term and the succession of remainders are the changing second term. The conclusion is obvious: the remainders are diminished without limit. But the remainders are called the remainders because they are left behind after the excesses have been removed. And, if they are diminished without limit as division is continued, it can only be that the excesses, when taken without limit, exhaust the magnitude being divided. Is it possible for the excesses to consume more than the whole through such division? No. At each stage of division only part of what remains is removed. Therefore the statement is proved.⁸

The statement just proved differs subtly from what was asked originally, and necessarily so. In the opening question and in Campanus's comment the remainders of division and their progressive diminution stood at the focus of concern. In this statement, however, concern has been redirected toward the whole magnitude and its eventual consumption.

To show the tendency of a series of magnitudes requires that the members of the series be successively compared with an invariant quantity of the same kind. In the case of continued division, the single possible standard of comparison is the magnitude assumed at the start of division, since it is the only invariant quantity to which the remainders themselves might be related by means of the defining proportion or any of its modifications. When the remainders of division are considered in isolation, as they were in Campanus's note and in the original question, a proof of their tendency is not even to be hoped for because the single standard against which they might be measured has been excluded from view. The results of division had to be restated to include the continued presence of the whole magnitude before a proof could be offered.⁹

In this shift of focus from continuous diminution to eventual exhaustion can be seen a shift in conceiving the process of continued division. When continued division is viewed as continuous diminution, the results of division make up a series of magnitudes known only in relation to each other. Viewed as eventual exhaustion, on the other hand, what before was considered a process of constant subtraction ending in either an infinitesimal particle or none at all, can now be seen as an extended effort of addition which produces a known sum. The parts which before were simply thrown away, the excesses, can now be seen to fill in the divided magnitude. Because the unending process has been placed within a known context and because its tendency can be stated in terms of an already established result, its inherent lack of definition has been tamed and it gains an intelligibility, albeit an indirect one, it did not formerly possess.

If the *Quaestiones* is recognized as a dialogue between master and student, it is only natural to expect ideas introduced in the first question to be explored further in the one that follows. In the second question the counterpart to the first is taken up.

As a consequence, it is asked whether an addition can be made to a magnitude according to proportional parts [applied] without limit. (3. 33-34)

To one who has just learned the lesson of the previous question, the possibility of what is entertained here seems slight. Thus the student maintains:

If [such an addition] could [be made] it would follow that a magnitude would be augmentable without limit....The consequence follows [from the antecedent], for from the fact that addition happens without limit, the magnitude will be augmented without limit, since it is added on to through the addition. (4. 1-7)

Diminution without limit was conceived as the achievement of a finite sum through an endless addition of excesses. But that demonstration was only possible because the addition, although endless, always took place within a predetermined boundary and could be shown never to leave it. In this case, however, because the parts are added first on to the extremity of the magnitude and then on to each other, they successively establish and reestablish the boundary of their sum. And, since the parts are to be applied without limit, it seems obvious that the magnitude will not have a fixed extremity.

In the first question the master responded to the student by fabricating an alternate understanding of part and division. In this question, he responds by sketching the mechanics of endless addition as they might appear from an alternate point of view:

It is argued the opposite way that whatever can be removed from some magnitude can be added to another. But subtraction from some magnitude can take place according to [proportional] parts without limit, in this way therefore, it can be proved that [a magnitude] is augmentable without limit. (4. 8-11)

The student views the parts to be added only in relation to one another and so is at a loss to infer the boundary they might approach in sum. The master counters this perception of addition with another. In some cases, the addition to one magnitude by proportional parts can be seen in terms of the exhaustion by proportional parts of some other magnitude. By presenting addition in this way, the master suggests the existence of a boundary that might be established *independently* of the process of addition and thus also suggests that the process itself might be seen as the eventual exhaustion of a predetermined quantity.

The qualification in what was just said was "in some cases." The different cases of addition according to proportional parts are distinguished according to the ratio in which addition is made:

For the sake of the question it must be noted first what the ratio of equality is and it is [the ratio] between equals. Another [kind of ratio] is the ratio of greater inequality which is the ratio of the greater to the lesser, such as four to two. Another [kind] is of lesser inequality which is of the lesser to the greater, such as two to four...And it follows from this, that addition to a quantity can take place in three ways. (4. 19-25)

When the addition is made without limit by parts applied in a ratio of either equality or greater inequality, the resultant "whole" will be unbounded, as is clear. When addition is made with parts applied in a ratio of lesser inequality, the claim is made that the "whole will never be unbounded." The reason offered to support this conclusion is simple, and unfortunately, almost totally obscure:

This is because the whole will have a fixed and definite ratio to the [magnitude] assumed at the start, that is, to that [magnitude] to which addition is made. (4. 30-31)

It is the task in what follows to reconstruct the understanding that lies behind this statement.

The question's "first conclusion" provides a point of departure:

The first conclusion is that if a quantity of one foot be assumed and addition be made to it without end according to the subduplicate ratio, so that first one half of a foot is added to it, then a fourth, then an eighth, and so on without limit, doubling the subduplicate, the whole will be exactly twice the [magnitude] assumed at the start. This is clear since if from some [magnitude] parts are taken away according to this order, exactly double of what is removed first will be removed ultimately from the whole, as is clear from the first, that is, the preceding question. Therefore the same argument applies if the parts are added [to a magnitude]. (5. 9-15)

In this example of continued addition can be seen the fulfillment of the master's earlier suggestion. The parts added to one magnitude are understood as parts subtracted from another. It is as if the two magnitudes were butted up against one another and the process of addition reclaimed the latter magnitude into the whole. In this case, to determine the size of the magnitude to be exhausted in the course of unlimited addition is easily done. The fact that the subduplicate series composed on a unit basis exhausts the unit was presented as a corollary to the demonstration of the previous question.

What would be the result if addition were made according to the subtriplicate ratio? What magnitude could such parts be considered to exhaust? To answer these questions it is necessary to backtrack and to reconsider continued division.

When division is made according to proportional parts, the excess at each stage of division is less than the previous remainder by a factor of the ratio that governs how division is made. For example, when two-thirds of a unit magnitude is removed in division. Chart 1

removed in division, and then two-thirds of the first remainder is removed and so on, the excess at each stage is two-thirds of the previous remainder (see

	Excess		Remainder	
0)	0	0	1	1
1)	(2/3)(1)	2/3	(1/3)(1)	1/3
2)	(2/3)(1/3)	2/9	(1/3)(1/3)	1/9
3)	(2/3)(1/9)	2/27	(1/3)(1/9)	1/27

chart 1). Because of this, given any excess and the ratio of division it is an easy thing to calculate the size of the previous remainder.

A more complex relation exists between successive excesses. Each excess falls short of the one that precedes it by an amount equal to the preceding excess diminished by a factor of the governing ratio. Consider chart 1 again. There, the second excess, 2/9, falls short of the first excess, 2/3, by 4/9, and 4/9 is 2/3 taken 2/3 times.

When addition is made according to proportional parts in a ratio of lesser inequality, the addend at each stage of addition is less than the previous addend by a factor of the ratio that governs how addition is made. Thus, when addition is made in the subtriplicate ratio, the second addend is one-

third of the whole and each subsequent addend is one-third of the one it follows (see chart 2).

The difference between successive addends may also be expressed in another way. It can be said that each falls short of the one that precedes it by an amount equal to the preceding

Chart 2					
	Addends				
1)	1	1			
2)	(1/3)(1)	1/3			
3)	(1/3)(1/3)	1/9			
4)	(1/3)(1/9)	1/27			

addend diminished by some ratio and that this ratio is the same for every pair of consecutive addends in the series. Consider chart 2 again. In the subtriplicate series, the second addend, 1/3, falls short of the given magnitude by 2/3, an amount which can be understood as that magnitude taken 2/3 times. Similarly, the third addend, 1/9, falls short of 1/3 by 2/3, which can be understood as 1/3 taken 2/3 times. And the same is true for the rest.

Now the groundwork has been laid for the crucial insight. From the descriptions above, it is clear that the addends in one series bear to one another the same relation that the excesses bear among themselves in another series. In these examples, the addends of addition made in a subtriplicate ratio successively fall short of each other to precisely the same degree that each excess of division made in the two-thirds ratio is deficient of the one it follows.

Because of this, the addends of a process of addition performed according to one ratio can be understood as the excesses left behind by division performed according to another. The whole these terms eventually exhaust can be calculated as the "remainder" of the stage previous to the removal of the first "excess." And, as should be evident, that whole will also be the sum that the addends eventually approach. Consider chart 3. There, the addends in the subtriplicate series are expressed as the excesses of division

according to a ratio of Chart 3 two-thirds. The first excess is 1, the first addend, and so the 0) 0previous remainder, 1) (2/3)(3)which must when 2) (2/3)(3)of two-thirds yield 1, is 3) (2/3)(1)3/2. Therefore, since it 4) (2/3)(1)

Churt o					
	Excess		Remainder		
0)	0	0	3/2	3/2	
1)	(2/3)(3/2)	1	(1/3)(3/2)	1/2	
2)	(2/3)(1/2)	1/3	(1/3)(1/2)	1/6	
3)	(2/3)(1/6)	1/9	(1/3)(1/2)	1/18	
4)	(2/3)(1/18)	1/27	(1/3)(1/18)	1/54	

proved that the excesses of division according to proportional parts exhaust the divided whole, the sum of the subtriplicate series when taken without limit is three-halves the magnitude chosen to be the first addend.

The trace of this understanding can be seen in the first conclusion, in the justification given for the quantity declared the sum in the subduplicate series (p. 36). There, the crucial turn of argument was provided by the observation of the relation between the first excess and the whole that the series of excesses ultimately exhausts. The one was presented as calculable because the other was already known.

A more explicit manifestation of the same understanding can be seen in the second conclusion:

The second conclusion is that if some quantity, such as a foot be assumed, and then a third as

much is added, and afterwards a third of the addition and so on without limit, the whole will be exactly a foot and a half [long]. That is, it will be in the ratio of the sesquialtern to the magnitude assumed at the start. And with regard to this, the following rule should be recognized, that we ought to see by how much the second part falls short of the first and the third of the second and so on with respect to the others, and to call that [quantity] by its denomination and the ratio of the aggregate whole to the magnitude assumed [at the start] will be as the ratio of the denominator to the numerator. For example, in the case proposed, the second part which is one-third the first falls short of the first by two-thirds, therefore the ratio of the whole to the first part or to the [magnitude] assumed [at the start] is three to two and this is the sesquialtern. (5. 16-26)

The use of "rules" to encapsulate a method of calculation or a criterion for judgment is a commonplace in medieval academic texts. Unlike other rules presented by other authors, this one is not derived in the text itself. But, because of what has already been said about addition without end, the function of its procedures can be readily unpacked.

When seeking the sum of a series of proportional parts, the student is instructed "to see by how much the second falls short of the first and the third of the second...and to call that quantity by its denomination." The amount that each term falls short of the one preceding it is to be isolated and expressed in terms of the preceding one. In the example given, that of the subtriplicate series, 1/3, the second addend falls short of 1, the first, by 2/3 which is called "two thirds [of the preceding term]." The third falls short of the second by 2/9 which is again "two-thirds" and so on.¹⁰

In this step, the student, in effect, deduces from the series of addends what the governing ratio would be were the addends considered as excesses. The analogy between this procedure and the one laid out above for the same purpose is obvious. There, the governing ratio was discovered as the fractional multiplier used to diminish the first of two consecutive addends to yield the amount by which the second fell short of it. Here, the same ratio is discovered as the relation of the amount of difference in the first terms, a method which, though distinct from the former, is functionally equivalent to it.

Once this ratio has been distilled, it is put to use in the calculation of the eventual sum of the series: "and the ratio of the aggregate whole to the magnitude assumed [at the start] will be as the ratio of the denominator to the numerator." The ratio denominating the difference between consecutive addends is expressed as a fraction, inverted and taken in this form as the ratio between the whole that is finally achieved and the first addend.¹¹

As was shown above, in any process of division, each excess falls short of the previous remainder by a factor of the governing ratio. Or, to state the same situation another way, each excess is to the remainder previous to it as the numerator of the governing ratio is to its denominator. Thus, to calculate the aggregate sum of addends, which, on the analogy of division, is the remainder previous to the first excess (addend), requires that a quantity be found which will have to the first addend the ratio of the denominator of the governing ratio to its numerator.

The exposition just completed was taken up to explicate the following statement:

If, in turn, [addition] is made according to a ratio of lesser inequality, [the whole] will never be unbounded although the addition be made without limit. This is because the whole will have a fixed and definite ratio to the [magnitude] assumed at the start, that is, to that [magnitude] to which the addition is made. (4. 28-30) The meaning of this passage should now be clear. In every series of addends built up according to a ratio of lesser inequality, each term is some part of the term which precedes it. Because of this, every series of such addends (and only the addends of such series) can be interpreted as a series of excesses and the ratio governing the corresponding division can be deduced. Inverted, this newly discovered governing ratio becomes transformed into the "fixed and definite ratio" between some magnitude and the first addend. And the route to proving that this magnitude is the eventual sum of the addends is readily indicated by the shift in perception which allows the successive augmentation of the original magnitude to be considered as the eventual exhaustion of this new one.

The questions of treatises written in the *sic et non* style of the disputation each present a lesson in which a student is taught by a master. In these lessons, the master is responsible for the education of his student and strives to introduce him into the orthodox understanding of the discipline being studied. In part, orthodoxy is a mode of perception. An aspect of what it means to be educated in a tradition is to have adopted the established outlook on the world and to appreciate why it is to be preferred above any other. Orthodoxy is also a mode of communication. It is expected of educated people that they be able to speak with precision and accuracy, in terms acceptable to other educated people, about the matter at hand.

In the first two questions of the *Quaestiones*, Oresme has presented the master's effort to inculcate the orthodoxy of the infinite in his student. Within this orthodoxy, infinite processes are never seen in isolation but are always viewed in terms of a preestablished whole. The vocabulary in which the orthodoxy is couched is the language of proportion. Ratio and the relation of ratios form the means of describing the processes posited and of demonstrating their eventual results. Within philosophy, ethics, and theology, understanding makes up a whole and until it has been fully grasped, each aspect of it remains provisional. In the case of the student's appreciation of the infinite, this characteristic is clearly visible. As a beginner in mathematics, the student has only the crudest grasp of ratio and proportion, the means the master employs for denomination and proof. As a consequence, he must accept the basis of the proof of the first question in the form of a supposition and the teaching of the second question as a rule. The student's knowledge is contingent and will continue to be so until the sophistication of his understanding matches that of his master's.

Oresme's willingness to present a mathematical subject in a treatise written *sic et non*, his willingness to explore a mathematical subject dialectically rather than deductively, gives some insight into his notion of mathematical understanding. To Oresme, mathematical understanding seems to have been of the same kind as the understanding of the traditional disciplines of disputation. It was not a special sort of knowing to be differentiated from all others.

Notes

¹ Nicole Oresme, *Quaestiones super Geometriam Euclidis*, edited by H. L. L. Busard (Leiden, 1961), p. x. See also A. Maier, *An der Grenze von Scholastik und Naturwissenschaft* (Rome, 1952), pp. 270, 345. Busard's text has been reviewed and extensively emended by J. E. Murdoch in *Scripta Mathematica* 27:1 (1964), pp. 67-91. An emended version of the Busard text of questions 10-15 appears with translation, notes and introduction as Appendix 1 of M. Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions* (Madison, 1969), pp. 521-575. My translation of the questions under study appears as an appendix.

² Two editions of Campanus's redaction of the *Elements* have been used in preparation of this essay. They are E. Ratdolt's *Praeclissimus liber elementorum Euclidis* (Venice, 1482), hereafter called Ratdolt; and J. Lefevre's *Euclidis megarensis geometricorum elementorum* *libri XV. Campani Galli transalpini in eosdem commentariorum libri XV*, etc. (Paris, 1516), hereafter called Lefevre.

Previous study of this portion of the *Quaestiones* is confined to Murdoch, pp. 68-70 and Clagett, pp. 130-31; 508-10. Clagett also considered infinite progressions and their sums in other works of Oresme and his contemporaries, see pp. 495-508. An old chestnut on this latter subject, cited by greater scholars than I, is H. Wieleitner's "Zur Geschichte der unendlichen Reihen in christlichen Mittelalten," *Bibliotheca Mathematica*, 3 Folge, 14 (1913-14): 150-68. See esp., 150-54.

³ The Latin adjective *infinitus* is the composition of *in* + *finio* and so has a primary sense, when used with respect to quantity, of "boundless" or "immense." The English cognate "infinite" possesses positive overtones as descriptive of a quality that is something in its own right; in it the privative has lost much of its immediate connotative impact. Because of this, every attempt has been made to avoid its use as the translation of its Latin ancestor. Only in those cases in which avoidance would have made a complete farce of the fluency of translation (almost all of these being uses of the adverbial form) has "infinite" been deemed acceptable.

⁴ Busard, p. 1, lines 5-8. All subsequent textual references are by page and line numbers and are included in the body of the essay. The word *proportio* has been translated "ratio," while the word *proportionalitas* has been translated as "proportion." As is clear, *proportionale* has been translated as "proportional."

⁵ Ratdolt, 2v. See also Lefevre, 4r.

⁶ Note that the Euclidean definitions of part and ratio (*Elements*, Book 5, Definitions 1 and 3) do not focus on this aspect. Because of this, they were passed over in preference for Definition 9.

⁷ Because this is a supposition, the effort has not been made to substantiate it. Nevertheless, it is credible. On the assumption that *a* is greater than *b*, the ratio a : b added to itself results in a greater ratio, $a^2 : b^2$. And with continued addition of the same ratio, it continues to increase. Thus a : b taken three times is $a^3 : b^3$, four times $a^4 : b^4$, and so on. The same ratio added to itself without limit would seem, in this way, to produce a ratio inassignably large. ⁸ For those who might have found the foregoing demonstration unsettling, the following derivation of an equivalent statement is offered as a tranquilizer.

For any geometric series $a + ab + ab^2 + ... + ab^{n-1} = Sn$, Sn = a($(1-b^n) / (1-b)$) as can be confirmed by division. If with respect to a unit magnitude *a* is the first part removed, the "first excess," and *b* the first portion left behind, "the first remainder," then the sum of the excesses will be *a* ($(1-b^n) / (1-b)$) where n + 1 is the number of divisions made. Since |b| < 1, $\lim_{n\to\infty} b^n = 0$ and the sum of the excesses after division has been continued without bound will be a (1 / (1-b))) = 1, that is, the continued removal of excesses will exhaust the magnitude. Note that in this derivation, as in the one presented in Oresme's text, the sum of the excesses is only calculable because the remainder b^n can be evaluated as zero as division is made without limit. The crucial difference between the approach used here and the one of the text is found in the difference between the ways used to evaluate the remainders.

⁹ A comparison with the modern derivation, while not particularly helpful in this context, is at least interesting. The modern analyst employs two modes of mathematical description in the course of his exposition. First he describes the process of continued division according to proportional parts with a geometric series. Then, using a completely different apparatus, one formulated without regard for this particular series, examines his description for the symptoms of convergence. This latter mode of description, the interval language which supports the limit concept, allows the analyst to make the type of comparison mentioned in the body of the essay. It is a means which may be used willfully to erect standards of comparison. With it, the analyst can import his own fixed quantities into a mathematical context.

¹⁰ Murdoch, p. 69. Although Murdoch interprets this part of the passage correctly, he goes on to add that "though Oresme does offer a proof of his summation of [a] series [of excesses], he gives no inkling of one when it comes to his rule" (p. 70).

¹¹ Apparently Oresme wrote ratios in the bilevel form used today as the fractional notation, see E. Grant, "Part I of Nicole Oresme's *Algorismus proportionum*," *Isis*, 56 (1965): 328-29.

Appendix

Question 1

Concerning the book of Euclid it is first asked about the dictum of Campanus in which he laid down that a magnitude decreases without limit. It is sought first whether a magnitude decreases without limit according to proportional parts.

And it is first argued that it does not. There are not an unlimited number of parts in a continuous [magnitude], therefore neither is there an unlimited number of parts of the same ratio. The antecedent is obvious, since otherwise [the magnitude] would be without limit. The consequent is obvious, since any proportional part is a part of the same quantity as any other [proportional part of that magnitude], therefore the parts of the same [magnitude] are of the same ratio and of the same quantity.

According to Campanus in the commentary [the matter] appears in the opposite way.

For the sake of the question it must be noted: first, what is meant by proportional parts or parts of the same ratio; second, in how many ways such parts can be imagined; third, how something can be divided up into such parts; fourth, suppositions and conclusions.

First, it must be noted that parts are called proportional with respect to a continuous proportion; and that such proportion is a likeness of ratio, as is said in the commentary to the ninth definition of the fifth [book], where it is said that a [proportion] is held between at least two ratios; and for this reason Euclid said that the minimum number of terms in which [a proportion] is found is three and the maximum number cannot be given, since it goes on without limit. From this it follows that properly, one does not speak of a proportional part or of two proportional parts, but that there should be at least three [proportional parts], and there could be an infinite number. And [such parts] are called continuously proportional when as the first is related to the second, so is the second to the third and so with the rest if more are assumed.

Second, it is answered that division according to such proportional parts occurs in as many ways as there are continuously proportional [parts] and there are as many of these as there are ratios, which are without limit. For example, it could be that the first is twice the second and the second twice the third and so on, just as it is commonly said of continued division; and it could be that the first is three times the second and the second three times the third and so on.

Third, it is said that a line and any continuum whatever can be divided up into such parts. A line can be divided up in two ways because it has two ends and such parts can start from either one. A surface can be divided up in an unlimited number of ways and the same [is true] of a body.

Fourth, the first supposition is laid down as the following: that if some ratio becomes augmented without limit, the first term remaining unchanged, the second term will become diminished without limit. This is clear since the ratio between two [terms] can be increased without limit in two ways, either through the limitless augmentation of the first term or the limitless diminution of the second term.

The second supposition is that if to any ratio another [equal to it] is added and then another and so on without limit, that ratio will be augmented without limit and this applies to all quantities.

The third supposition is this, that addition can be made according to proportional parts to any quantity and from the same quantity diminution can occur according to proportional parts.

The first conclusion is therefore that, if some part is removed from a quantity and from the first remainder the same part is removed and from the second remainder the same part is again [removed] and so on without limit, the quantity, through this manner of endless subtraction, will be exactly consumed, neither more or less [than the whole of it will be removed]. This can be proved thus: The whole assumed at the start and the first remainder and the second and the third and the rest are continuously proportional (as can be proved by arguing from a transformed ratio), therefore there is, in this case, some ratio and as much again and so on without limit. Thus it follows from the second supposition that the ratio of the whole to the remainder increases without limit since it is composed with itself. And one term, that is the whole, is imagined not to change, therefore, according to the first supposition, the remainder is diminished without limit. And therefore the whole of the quantity is precisely consumed.

From this the corollary follows that if from some [magnitude] a foot long half a foot is removed and then half the remaining quantity and then half of the next remainder and so on without limit, precisely a foot will be taken away from it. The second corollary is that, if from some [magnitude a foot long] one thousandth part of one foot is removed and then a thousandth part of the remainder of that foot and so on without limit, exactly one foot will be subtracted from it.

But this is doubted: Since exactly half a foot, half of the remainder of that foot and so on without limit make up one foot, let this whole be a. Similarly, according to the second corollary, a thousandth part of a foot and a thousandth part of the remainder and so on without limit make up one foot, let it be b. Thus it appears that a and b are equal, but it is proved that they are not: Since the first part of a is greater than the first part of b and the second part of ais greater than the second part of b and so on without limit, the whole of a is thus greater than the whole of b.

And this is supported [in this way]: If Socrates is moved over a during an hour and Plato over b, and if they divide the hour according to proportional parts and correspondingly move over a and b, then in the first proportional part [of the hour] Socrates will be moved faster than Plato and similarly in the second and those beyond. Therefore, Socrates will move through more space than Plato, and therefore a is a greater space than b.

In reply to this argument the antecedent is denied, namely, that the first part of a is greater than the first part of b and so on, for the reason that, although the first part of a is greater than the first part of b and the second part of a is greater than the second part of b, nevertheless [when the parts are taken] without limit, eventually one [part of a] is arrived at which will not be more than its counterpart [in b], but less.

From this the response to the question is clear, that any continuum can have proportional parts without limit. In imagination and likewise in reality, the first can be separated from the others by a process of corruption, and then the second, and so on without limit.

In reply to the arguments advanced in opposition I deny the consequence, namely, that any part would be of the same quantity as any other and for a proof of it I say that although some proportional part may be a part of the same quantity as others, nevertheless it is not the same in quantity as them since it is of the same ratio. They are not equal for then it would follow that all would be equal among one another.

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Question 2

As a consequence it is asked whether an addition to a magnitude can take place through proportional parts [applied] without limit.

First it is argued that it cannot. If it could, it would follow that a magnitude would be actually augmentable without limit. This consequence is contrary to what Aristotle said in the third [book] of the *Physics* and to what Campanus said in the first [book] of this [work], where he laid down the difference between magnitude and number, namely, that number increases without limit and does not decrease and that for magnitude the opposite is true. But, the consequence follows [from the antecedent], for, from the fact that the addition happens without limit, the magnitude will be augmented without limit, since it is added on to through the addition.

It is argued the opposite way that whatever can be removed from some magnitude can be added to another. But subtraction from some magnitude can take place according to such parts without limit, in this way, therefore, it can be proved that [a magnitude] is augmentable without limit.

An example may be offered of a right triangle and an acute angle, or of two right angles. Let there be a line on top of another which makes two right angles which are a and b. Then a line, c, turns toward one extremity, d. It is now argued thus: as much as angle b decreases through this motion, so much angle a increases. It is clear that whatever is removed from angle b is added to angle a. But angle b decreases doubly, threefold, fourfold and so on without limit, therefore angle a increases without limit.

For the sake of the question, it must be noted first what the ratio of equality is and it is [the ratio] between equals. Another [kind of ratio] is the ratio of greater inequality which is [the ratio] of the greater to the lesser, such as, four to two. And another [kind] is of lesser inequality which is of the lesser to the greater, such as two to four. And these names differ according to the relation of position and supposition as is clear in what has been said. And it follows from this that addition to a quantity can take place in three ways.

Second, it must be noted that if addition be made according to proportional parts [applied] without limit in either a ratio of equality or [one] of greater inequality, the whole will be without limit. If, in turn, it is made according to a ratio of lesser inequality, [the whole] will never be unbounded although the addition be made without limit. This is because the whole will have a fixed and definite ratio to the [magnitude] assumed at the start, that is, to that [magnitude] to which the addition is made, as will be explained later.

Finally, it must be noted that each [magnitude which is] less than another, but bears a fixed ratio to the other, is called in relation [to the other] either the fraction or fractions of it or the part or parts [of it]. And this is clear in the definitions of the seventh [book] of Euclid. And the lesser is named by the two numbers, of which one is called the numerator and the other, the denominator, as is clear in the same [text]. For example, one is less than two and so it is called with respect to two one half, and with respect to three one third, and so on. And two is called with respect to three twice one third, and with respect to five twice one fifth. And they ought to be written in this way, the two is called the numerator and the five is called the denominator.

The first conclusion is that if a quantity of one foot be assumed and addition be made to it without end according to the subduplicate ratio, so that first one half of a foot is added to it, then a fourth, then an eighth and so on without limit, doubling the subduplicate, the whole will be exactly twice the [magnitude] assumed at the start. This is clear since if from some [quantity] parts are taken away according to this order, exactly double of what is removed first will be removed [ultimately] from the whole, as is clear from the first, that is, the preceding question. Therefore the same argument applies if the parts are added [to a magnitude].

The second conclusion is that if some quantity, such as a foot be assumed, and then a third as much is added, and afterwards a third of the addition, and so on without limit, the whole will be exactly a foot and a half [long]. That is, it will be in the ratio of the sesquialtern to the magnitude assumed at the start. And with regard to this, the following rule should be recognized, that we ought to see by how much the second part falls short of the first and the third of the second and so on with respect to the others and to call that [quantity] by its denomination and the ratio of the aggregate whole to the magnitude assumed [at the start] will be as the ratio of the denominator to the numerator. For example, in the case proposed, the second part which is one third of the first falls short of the first by two thirds, therefore the ratio of the whole to the first part or to the [magnitude] assumed [at the start] is as three to two, and this is the sesquialtern.

The third conclusion is this, that it is possible that an addition be made to some quantity according to unproportional ratios of lesser inequality and the whole will be unlimited [in magnitude], notwithstanding. But if this be done proportionally, the whole will be bounded as has been said. For example, let the quantity assumed be of one foot, to which half of a foot is added in the first proportional part of the hour, then a third in the next, then a fourth, then a fifth and so on without limit according to a succession of numbers. I say that the whole will be without limit, which is proved in this way. There exist in this case an unlimited number of which any one will be greater than one half, therefore the whole will be without bound. The antecedent of this argument is clear since a third and a fourth are more than a half and similarly [the terms] from a fifth to an eighth are greater than ds on without limit.

In response to the arguments raised in opposition: "it follows magnitude" and so on, it can be stated that a magnitude will be augmented without limit. One sense of "without limit" can be with reference to the act of augmenting and in this sense it can be conceded that such an actuality can come about in an unlimited number of ways, as long as [the augmentation] is continued. But this is an improper sense [of "without limit"] and it follows rightly from the question that another sense is proper. Because [to say] that it will be augmented doubly, quadruply and so on without limit is false and does not follow from the question.

In response to the other argument, namely, when it was argued about the angle that "it will be augmented as much" etc. I say that there is a need for a distinction here since "as much" and "so much" can denote an arithmetical ratio which extends to the extent of the quantity of the excesses; in such a case, I concede the major premise [that a will be augmented by as much as b is diminished] and deny the minor since the supposed solution does not occur [namely, that neither a nor b suffers change without limit]. Alternately ["as much" and "so much"] can denote a geometrical ratio and in this case, I deny the major since a will not be augmented in such a ratio as b is diminished, but it is required that a will be augmented by as much as b is diminished. And it can be argued in this way concerning any other quantity or quality and this is clear since when the angle b is diminished doubly, the angle a is not augmented doubly, rather it is said to be augmented doubly when c turns all the way down to d and that will be when b is fully diminished and corrupted. And thus the response to the question is clear, so ends the question.



Toast to the Senior Class May 12, 2000

Leo Raditsa

To say farewell, I would like to tell you: You are teaching me—a much harder thing than teaching you. It started the day I walked into a classroom thirty-five years ago at NYU. For a long time I fought it.

Coming across takes two. To accept is harder than giving. And unless there is movement in both directions, little confidence lives. For a teacher, ceasing to hang on to his knowledge takes years.

You give me a lot also in the things you don't say, in your obvious tact, in your experience in putting up with things you cannot do much about. I mean your savvy, your readiness to put up with imperfection in your work and your teachers, and, above all in your readiness to go on—I always fear I might stop. Most of all, your sense of humor, which amounts to saying you are really there and not living in distraction, shows me something I do not readily see.

I only began to understand this undoing in me, that I was beginning to lose my fear of students, when memories of my teachers crowded upon me: actual faces and voices. I grew astonished at the number of very great teachers I had had. I had had little sense these paintings lived in me. They came to life in me when I let go of what these teachers had given me. Dealing with you slowly brought me to move on my own. So

Leo Raditsa, tutor at St. John's College since 1973, was editor of *The St. John's Review* from 1978 to 1982. His essay, "The Collapse of Democracy at Athens and the Trial of Socrates," is printed below.

as I toast you in a minute, in my heart I will also toast my teachers, and the ragged world I now realize they knew I would live in. In this way the present lives the past's strength.

For you, too, what you give your teachers may be the more important way you learn from them, for this giving means you are strengthening your wings. So I toast these beginnings in you which may not be yet obvious to you, and maybe not to us.

Your capacity to understand you are students impresses me, your savvy and sense of solidarity with your peers, your readiness to converse with each other, the prerequisites for independence, and your readiness to respect your teachers, and yet your ability to size them up, to live with their defects, to be truly, not defensively, critical of them. In short your willingness to make the best of it. You also teach me a sense of measure—something that cannot be learned in books. I mean proportion where all living beauty hides....

I wish you fortitude and endurance in these beginnings, and also the courage to change your minds, and above all the courage to listen to your inner voice, for nobody can really make those choices except yourself. When you feel fright before those choices, when you wake up and wonder whether you have an education, it is a sign that your education has taken hold: You are on your own. Without standing being lost, there is little chance of beginning. I wish you also the courage of going through the eye of the needle—not all the time but sometimes—of daring to be true to your feelings, I wish you the capacity to take risks and to take disappointment.

More importantly, I have learned from you that the only virtues you can learn are the ones you see before your eyes, not the ones you read of in books. They cannot be read alone. You help me read the book of life.

I raise a toast to you from us your teachers, to your future, and to teaching as a two-way street, something you teach me.

The Collapse of Democracy at Athens and the Trial of Socrates Leo Raditsa

Thucydides did not finish his account of the "intense movement" (so he named it) among the Greek peoples that he judged to be the greatest event of history including the Trojan War. The incompleteness of Thucydides' account suggests it never ended—and perhaps there is some truth in that. For the kind of war—and in his opening paragraph he carefully defines it—Thucydides describes, without specific political aims and which proceeds by revolution, is difficult to end. One can terminate hostilities, but to make peace: that is another, much more difficult matter.

The crisis which we call the Peloponnesian War did, however, come to some sort of end and it is about that end and what came after it, especially the trial of Socrates, that I am going to talk to you tonight. The period runs roughly from 410 to 399, the year of Socrates' trial.

The historical question I wish to face is what the relation is of the trial of Socrates to the collapse of democracy which occurred at Athens with the slow ending of the war. To put it simply, why was Socrates prosecuted in 399 instead of some time earlier, for instance, in 423 when Aristophanes had *The Clouds* produced?

Xenophon, who begins his narrative about where Thucydides leaves off, does not mention the trial of Socrates, although he does mention Socrates' attempt when he was in Prytany to prevent the illegal trial of the generals who had commanded at Arginusae in 406. Diodorus Siculus mentions the trial, but only in passing, the way he mentions the death of Sophocles in 406. I think ancient historians did not include the trial of Socrates in their compositions because they understood history to deal with the public life of a city, of its

This lecture was delivered at the Annapolis Campus on February 18, 1977.

officers and of its citizens in public assembly and in battle. They did not conceive history to include the relation of private to public life, something which was the subject of much of Socrates' activity. Although Socrates was charged with a public crime-graphe, not a dike, which referred to a civil suit, as Socrates reminded Euthyphro at the start of his conversation with him-he was charged as a private citizen, not as an office holder.

There was another and deeper reason for not including the trial of Socrates in the ancient accounts of the period. In contrast to Plato-and in this he is profounder than Plato-Xenophon admits that he does not understand how it could have happened that Socrates was tried and condemned. That is, Socrates made him question the world his eyes saw, and this involuntary questioning is Xenophon's greatest tribute to Socrates. But this questioning did not extend to history. For Xenophon, history bore some relation to tragedy. But public men and cities suffered tragedy. To include the trial of Socrates in his composition Xenophon would have had to conceive of the tragedy of a private man. He could not—like most Athenians.

Think on it a second. All the Athenian tragedies are about public individuals, kings and princes, when they are not about Gods. There is something radically wrong with the way we read tragedies, as if they were about the lives of private individuals. The private individuals, the individuals who hold no office, appear in comedies. There they trip over their fantasies, which they take for actions, grow embarrassed at themselves, at the greatness they feel trapped in their insides but which betrays them when they open their mouths. There they grow haughty with their magnificent and outrageous gods. It is a measure of what happened at Athens that a generation after he had been subject to a comedy, Socrates became protagonist of an event that the best of his contemporaries knew they could not understand.1 For it was the tragedy of a private man. Even now we cannot easily integrate the trial of Socrates into the history of Athens and of the other Greeks, just as historians of the Roman empire hardly ever include the trial of Jesus in their accounts of that period.

The Collapse of Democracy at Athens

RADITSA

The last years of the war [and following years], the period from 411 to 401, represent the precipitation of that crisis in leadership which we call the Peloponnesian War. It is the period of the war in which the war became more and more something that happened to Athens and something that Athens did to herself. It is also the period in which Sparta took to the sea and in which Persia became increasingly deeply involved.²

The events of 411, the formation of the oligarchic government of the Four Hundred and then of the Five Thousand, which represented a reaction to the Sicilian disaster, not only shook Athens' domestic political confidence. They isolated Athens in the Greek world. The oligarchic revolutions in other allied cities which had accompanied the changes at Athens in 411 had not served, as the oligarchs at Athens had expected, to make settlement with the Lacedaimonians possible, but had instead contributed to bringing these cities under Lacedaimonian sway. Everywhere there was instability, and the cities lived on the brink of civil war. At Athens itself the situation was tense-the democracy had passed strict laws encouraging the punishment of those who had been involved in the oligarchic movement of 411. There were many exiles. The division which had occurred with the coming of the oligarchs in 411 had not been overcome. In an important sense Athens in 410 was no longer one city but two. This meant nobody knew what might happen next.

With the weakening of the predominance of Athens and her instability, other Greek cities grew more aggressive in their views. For the first time during the Peloponnesian War, Greek leaders, especially the Spartans, reckoned with public opinion outside of their cities. For instance, Pausanias, one of the kings of Sparta, is said to have intervened in the Athenian

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civil war at the end of the period of the Thirty because he feared the consequences to the reputation of Sparta if the slaughters of the Thirty continued.

In the first part of this period, the six years leading up to the destruction of almost the entire Athenian fleet at Aegospotami in September 405, the war was largely at sea for both sides. The sea war of these six years took place mainly in the Hellespont and in the Bosporus, and along the adjoining coasts of Thrace and Asia Minor with its three major islands, Lesbos, Chios and Samos. It was through these straits that many of the Athenian grain ships sailed. When she challenged Athens in this area, Sparta was aiming at her life lines, but not, in the beginning, at least, for total victory. For after several of the major battles she attempted to negotiate with Athens. For the first time in the war Athens was on the defensive in a way she had never been when Sparta had wasted Attica in the first years of the war.

For her part Sparta appeared to be without a coherent policy in this period. Her most noble commander drowned at the battle of Arginusae; Callicratidas tried to keep free of Persian entanglements; but Lysander, the Spartan commander who was to bring the war to an end, had no scruples about taking all the money he could from Persia for building the fleet and paying its crews.

The main events of this period were the return of Alcibiades to Athens in 407; the victory of the Athenian fleet at Arginusae in 406 and the unlawful trial and execution of the generals of the fleet which followed upon it; Lysander's destruction of the Athenian fleet at Aegospotami in the fall of 405; and the collapse of Athens in the period 405-401, especially after the siege and surrender, in the fifteen months which run from [April] 404 to [May] 403, when the Thirty were in power.

Of these events the collapse of Athens or the time of the Thirty, as it is usually called, was the most devastating. The experience of Athens during this period left an indelible impression on the whole ancient world. People thought of it with the same horror as the men of Colonus looked upon the face of Oedipus. Sallust's *Caesar*, written during the death agony of the Roman Republic, in the face of the procriptions of the young Octavian, recalls the horrors of the years of the Thirty at Athens with a vividness which makes one imagine Sallust had lived through the time. The Thirty, who were led by two of his closest relatives, and Socrates' trial—these were the two central experiences of Plato's life.

Somehow no matter what she did Athens always wounded herself. This is the terrible sense of this last decade and earlier—for it really started at Melos in 416. When the Athenian people illegally condemned commanders they suspected to be innocent after the great victory at Arginusae in 406, they hurt themselves. As Socrates later pointed out, when they violated their own laws they discredited themselves, destroyed their public life and made themselves incapable of recognizing and standing up to their real enemies.

In Socrates' presence Athenians knew they were doing this to themselves. This is the meaning of Alcibiades' wonderful and terrifying remark that Socrates was the only man in Athens who made him feel ashamed. In Socrates' presence he could not fool himself—he knew that what he did somehow betrayed what he was. Alcibiades meant that Socrates made him feel alive. Socrates gave men something like the feeling you sometimes get from infants when they make you wonder how you have become what you are.

Alcibiades' return to Athens in 407, with his appearance before the council and the assembly and his election to position of Commander in Chief, made a deep impression on Athens. They saw him now almost like an outcast, like Oedipus, forced to live beyond the protection of the laws, his life always in danger, in Sparta and Persia. Here was the man who in his life, almost in his person, summed up most of the destructive and constructive actions of the years since 415: the castration of the Hermae and the parody of the mysteries

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(from which he was exonerated), the expedition against Syracuse, the Spartan fortification of Decelea in Attica, and the involvement of Persia in the war—and constructively and more recently, the prevention of civil war during the oligarchic crisis in 411 and the re-establishment of Athenian control of the Propontis and the Bosporus in 410.

When Alcibiades sailed into the Piraeus he waited cautiously, without disembarking, for his friends and relatives to escort him up to the city. To many, both rich and poor, democrats and oligarchs, he seemed like the one individual capable at the same time of overcoming the division which remained within the city and of prosecuting the war with intelligence.

But something like six months later he was either not reelected or removed from his command because a subordinate—against his express orders—engaged Lysander and lost fifteen Athenian ships. He went into exile on his estate in the Chersonese. The great expectations had come to nothing the crisis continued.

Almost a year apart, the two great naval disasters at Arginusae and Aegospotami were in a sense both self-inflicted. I call Arginusae a disaster even though it was an Athenian victory, because its repercussions at home did much to discredit the unstable democracy. When Athens learned that the Spartan commander Callicratidas had encircled the Athenian commander Conon at Mytilene she sent out a hastily-gathered fleet of 110 ships which she manned with free men and slaves (who were later awarded their freedom).

Immediately after the Athenian victory a storm suddenly rose which prevented the Athenian commanders from picking up the several thousand dead and survivors floating in the rammed and waterlogged ships that had not sunk. At Athens, the news of the losses blunted the joy of victory. Following a little after the news, the Apaturia, a festival which drew together families to acknowledge births and marriages, made the grief worse. The matter came up in the council and the assembly. Under the influence of their politicians the people seemed unable to accept that some things are not under human control, that a storm occurs in "divine necessity," as one speaker put it.

RADITSA

Their politicians dominated them by nourishing their yearning to make someone responsible for everything. Theramenes, an important and able politician who had been a subordinate commander at Arginusae, accused his superiors of neglect. There was debate both in council and assembly, and the six generals who had dared to come back to Athens defended themselves ably and with witnesses, even though they had not yet been formally accused. At the point when it appeared the generals would win some kind of release the assembly was adjourned.

In a subsequent assembly it was proposed to vote on the guilt of the generals as a group and to count their previous testimony as a trial—all highly illegal. A brave speaker in the assembly attempted to stop the proceedings on the grounds of unconstitutionality (the *graphe paranomon*); but the people, turning into a mob, threatened him. This was the first, crucial attempt to resort to the *graphe paranomon* since it had been restored after the Four Hundred had abolished it in 411.³ It failed. But the grounds of the illegality had been clearly stated in the assembly. The crowd also intimidated all of the council except Socrates when it sought to keep the motion for sentencing from the assembly. The generals were condemned as a group and immediately executed.

Sometime later the people regretted their action, as they had been warned they would in the assembly. They turned upon their leaders and prosecuted them, depriving one of them of fire and water. But it was too late. All along they had known what they were doing was wrong, but they could not stop it. Against the speaker who had opposed them they had shouted that it was unthinkable that the people should not be allowed to do whatever they desired. After Arginusae the tension in many of the cities increased. Returned as acting Commander of the Spartan fleet, Lysander, with headquarters at Ephesus, supported the so-called oligarchs in a bloody seizure of power at Miletus. Four hundred of the wealthy and prominent citizens of Miletus were executed in the market place. For his predecessor Callikratidas's attempt to lead the Greek cities with words, Lysander substituted terror, for which party labels were mere pretexts. In Karia, a city allied to Athens was wiped out.

Sailing from Samos, the Athenian fleet found no support among the Greek cities. Except for Mitylene, all Asia turned away from Athens. When news came that Lysander was retaking Lampsacus on the Hellespont almost the entire Athenian fleet, one hundred and eighty ships, sailed to Aegospotami, a barren stretch of beach just fifteen stades (a stade is 600 feet) across the water from Lampsacus. Despite Alcibiades' warning—from his estate on the Chersonese he watched the whole disaster take shape before his helpless eyes—the Athenian commanders remained in their exposed position and offered battle to the Spartans for four days. On the fifth day Lysander surprised the Athenians after they had disembarked and destroyed or captured more or less their whole fleet. It was the Fall of 405.

At Athens they prepared for siege: all the harbors except one were filled up, walls were repaired and guards put on them. The city sought a hasty and incomplete unity in the restoration of full citizen rights to those who had been partially deprived in the previous troubles. But they did not recall the exiles.

At Aegospotami in assembly with the allies of Sparta, Lysander executed one of the Athenian commanders, because he had been the first to break the international law of the Greek cities. He had hurled the captured crews of two ships of Sparta's allies from a precipice, and in the assembly at Athens he had supported a motion to cut off the thumbs of all prisoners of war and make them incapable of ever rowing again. Lysander also showed himself as the undoer of Athenian outrages: at Melos, Torone, Scione and Aigina he restored the remaining original inhabitants.

From Chalcedon and Byzantium on the Bosporus and elsewhere, Lysander set the Athenian garrison loose on the condition that they sail nowhere else but to Athens. He wanted to burden Athens with as many mouths as possible. Everywhere the Greek cities turned to Sparta.

But at Samos, the other port of the Athenian fleet, the democracy held, and again knew itself in the slaughter of prominent citizens. For the first time since before the Persian wars, Athens was cut off from the sea, closed in upon herself—Athens, whom almost ten years before, Peisthetaerus in the Birds had called "the city of the lovely triremes."

Throughout the whole winter and until April of the following year, 404, Athens and the democracy resisted—and people starved. There was an early attempt at negotiation in which Athens offered to accept Sparta's leadership in alliance, a situation that would have allowed her considerable independence. But at Sparta the Ephors insisted on tearing down part of the walls. In response the people at Athens forbade any motions concerning peace. Men grew convinced that any terms with Sparta meant the fate of Melos.

In this tense and dangerous situation Theramenes managed to persuade the assembly to let him find out from Lysander whether the Spartans wanted to destroy the long walls to reduce Athens to slavery, or simply as a guarantee of their good conduct. Theramenes remained with Lysander who was besieging Samos until with the worsening situation at Athens the assembly granted him power to negotiate.

The new terms which the Lacedaimonians and their allies offered were much harsher than the previous demands of the Ephors: Athens was to have the same friends and enemies as Sparta. (In our terms this meant Athens lost the capacity for an independent foreign policy.) She was to tear down the long walls. Her fleet was not to number more than twelve ships. The exiles were to return.

When Theramenes returned to the starving city with these terms, men crowded around him in fear—but in the assembly there was still some resistance to surrender. In acting as go-between between the Athenian democrats who desired to resist to the end and the probably undecided Spartans Theramenes had saved his native city from total destruction—or rather from destroying itself. To the intoxicating sound of flutes, Lysander had sections of the long walls torn down. The Spartans and the returning Athenian exiles, according to Xenophon, imagined that that day meant the beginning of freedom for the Greeks. It was April 404.

In the following month the Athenian assembly, in the presence of Lysander, voted to give thirty men the power to revise the laws and reform the constitution. The Thirty promised to make the city clean and honorable and to impel the citizens to justice and excellence. Plato, then twenty-four years old, and many others, perhaps even Lysias (a speech writer, son of Cephalus and brother of Polemarchus, who appear in the first book of the *Republic*), believed in them at first.

The commission delayed the reform of the laws, but appointed magistrates and council, and started to rule. Before the council and with public balloting, they tried and killed notorious sycophants, individuals who had used the threat of prosecution for extortionary purposes. Although illegal, these killings won wide consent among the citizens, because men felt they were justified.

Soon, however, Critias, a close relative of Plato and an interlocutor of Socrates, asked Lysander for a Spartan governor and garrison to support him in dealing with unruly and subversive elements. With Spartan troops behind them, the Thirty now began to kill all individuals who might oppose them, and whose property would furnish the money necessary for the support of the Spartan garrison. At these outrages many went into exile, including Anytos (who later instigated the prosecution of Socrates) and Thrasybulus, who was to lead the democrats. Megara and Thebes teemed with Athenian exiles despite Sparta's order forbidding any Greek city to receive them. (By January 403, when the Thirty left Athens for Eleusis, where they had exterminated the population, perhaps as much as half of the male population had left Attica.)

Among the Thirty themselves the outrages also produced opposition. Theramenes, who knew the distinction between a moderate oligarchy and terror, told Critias that they were now much worse than the sycophants of the democracy who had extorted money, but not killed for it. Critias answered, brutally, that changes of constitution required killing: "How do you think thirty can rule over many without terror?"

Critias now disarmed all the population except three thousand of the more wealthy. All except these three thousand could be arrested and executed without trial. As Socrates later pointed out in his own trial, Critias sought to dominate by involving as many as possible in his outrages. Under the swords of the Spartan garrison he compelled the Three Thousand to condemn the inhabitants of Eleusis to death. When he could no longer tolerate the free-spokenness of Theramenes, he made the council his accomplice in his death.

Sometime during the early winter of 404, Thrasybulus, with about seventy followers, took the border fortress of Phyle which overlooked the whole Attic plain to Athens. The Thirty immediately responded, but were repulsed in a minor skirmish. This minor setback shattered their confidence and showed that their cowardice matched their brutality.

Sometime after this Thrasybulus, now with something like seven hundred badly armed followers, took the section of the Piraeus called Munychia. There in pitched battle the men of the Piraeus, as they now came to be called, managed to defeat the Thirty and the Three Thousand. Critias, first cousin to Plato's mother, and Charmides, his uncle, were both killed. Mindful that the enemy dead were citizens, the men of the Piraeus did not strip their bodies. They sought instead to use their victory to shake the by now largely forced loyalty of many of the Three Thousand (especially those who had not committed crimes) to the Thirty. Shortly after the battle the Three Thousand removed the Thirty from office and elected twelve to rule. The Thirty and their followers fled to Eleusis. It was January, 403.

At this point Athens was no longer a living city but three factions, one in the city, one in the Piraeus, and one in Eleusis. From Eleusis the Thirty sent men whom they fancied ambassadors to Lysander, saying there had been a revolt of the mob at Athens and requesting his help. Intent on surrounding the democrats at the Piraeus, Lysander managed the appointment of his brother as naval commander and authorization for himself to hire mercenaries. But Pausanias, one of the kings of Sparta, alarmed at the thought that Lysander might turn Athens into his private possession, convinced the Ephors and the Spartan assembly-ostensibly to help Lysander, but actually to prevent the destruction of the men of the Piraeus. Pausanias's expedition, with the Spartan army, amounted almost to a reopening of the war. In fact, Thebes and Corinth refused to join, because they said Athens had not violated any of her treaty agreements. With Spartan authority to come to a settlement, Pausanias managed to negotiate an agreement in which both the oligarchs of the city and the democrats of the Piraeus agreed not to fight each other. At Pausanias's insistence they also agreed to return property expropriated under the Thirty to its owners. The constitution of the democracy was restored. It was probably August 403, fifteen months after the assembly had first elected the Thirty.

For the next two years Athens lived in fear of renewed attempts to undo the democracy. In 401, upon rumors that the Thirty at Eleusis were hiring mercenaries, the whole city took arms and went out to meet them. During the ensuing negotiations the men of the city killed the commanders from Eleusis and managed a reconciliation with their followers, with the help of their relatives and friends in Athens.

Either at this time or two years earlier, in August 403, when Pausanias negotiated the reconciliation, every individual in Athens swore not to bear grudges for anything in the past. This meant nobody could prosecute for offenses under the Thirty, probably including the expropriation of property which Pausanias had sought to undo. The agreement to forget did not cover the Thirty, "the twelve" who had committed their "executions," and several other categories. It was contractual and could only be enforced upon appeal from individuals in court. (Andocides, for instance, appeals to it in his speech "On the Mysteries" in 399, the year of the trial of Socrates.)⁴

Athens After The Thirty and the Trial of Socrates

The atmosphere in Athens after the Thirty was somewhat unreal. It had become a city that feared disturbances and feared itself. It also remained in an important sense two cities. When you spoke at Athens during this time you always addressed two audiences, the men of the city and the men of the Piraeus. In these years Lysias spoke directly to a deep sense of unease and complicity with terrifying events which must have prevailed among the majority of Athenians. For the heroes of Phyle and Piraeus had been few. Lysias understood the deep struggle for self-respect Athenians waged during this time. "The Thirty killed my brother," he says, "they even made it hard to bury him—I will not forget." "Then, under the Thirty, you were afraid," he tells the judges, "but now there is nothing stopping you from voting the way you desire, now there are no excuses."

Lysias attacked Theramenes, not distinguishing him in any way from Critias. Theramenes had betrayed the trust the people had shown him and brought the city down in starvation. Everything that had occurred in the assembly which voted authority to the Thirty in the spring of 404 had been arranged beforehand, secretly, between Lysander and Theramenes. The vote had not been freely taken. If the Thirty had not killed Theramenes the democracy would have had to—a remark that, in its inverted way, pays a deep compliment to Theramenes.

In all the violence the only obvious palpable tie that remained between the factions was the gods; to them the city now made appeal. Of the Thirty, Lysias said, "they wanted us to participate in their shame instead of the gods, to substitute complicity with them for our common relation to the gods." He also described the Thirty as men who believed their power to be firmer than the vengeance of the gods—something quite like the Melians had said to the Athenians.

When the men of the Piraeus addressed the Three Thousand after their defeat in January 403, they spoke first of their common gods. Immediately after Pausanias had succeeded in bringing peace between the factions, Thrasybulus went up to sacrifice on the Acropolis: he meant to reaffirm that Athens belonged to Athena, who lived on the Acropolis. Perhaps Euthyphro exemplifies this newfound, somewhat showy piety of Athens after the Thirty. It is full of unquestioning assurance—and yet at a loss for words.

With this piety there was a forced and unconvincing blustering patriotism. Andocides did not blush to compare the Athens of the year of Socrates' trial with the Athens of the Persian wars. Anytos showed the brittle, touchy confidence of these years when he takes "personal" (as we would say) offense at Socrates' observation (in the *Meno*) that the sons of the pillars of the community had not turned out so well. People yearned for conviction, but were incapable of it.

Socrates came from another world—the world of Athens and the Greeks before the Peloponnesian War. At its outbreak in 431 he was about 40, and already famous throughout at least the Greek world. Men came from as far as Cyrene to listen to him.⁵ This is the Athens of the fifty years between the Persian and the Peloponnesian War, the Athens that neither feared itself or others. It was a city that did not fear the unexpected. A city in which important things besides crime happened on the streets. In fact, to that street life and its casual encounters, to how one can live on the streets, Socrates is one of our greatest witnesses. I think his refusal to wear sandals speaks of his feel for that life and of his insistence on its importance.

Another witness to that street life is Herodotus, whose book, like the *Odyssey*, is also a book of manners. Although—or perhaps because—careful and cautious, Herodotus is confident and respectful of his readers' intelligence, of their capacity to think. Socrates has the same respect for the intelligence of the people he encounters. He could tolerate the movement of other peoples' minds (when they did actually move) and he knew that movement to be as unexpected as truth. That is why he preferred to talk, to listen as well as to speak, rather than to write or teach.

Unlike Herodotus, Socrates did not travel—as he remarks, he never left town. Even when everybody went to a festival, he remained behind with the cripples and beggars in the deserted silent city. Herodotus instead went everywhere with the same ease that Socrates stayed home. Both give an example of the best kind of courage, the unassuming kind, the kind that does not have to prepare a face to meet the faces that you meet.

The Athenians of that time were used to living in a world that strengthened them, in a world where the throbbing glow of the sky was palpable, in a world that knew nuance, that could see the shape of the human body because it knew it to be more than the sum of its parts. Pericles says Athens was largely free of the jealousy of the lives of others which contributed so much to the later hatred of Socrates. In the presence of Herodotus and Socrates one feels one's pretensions like a kind of awkwardness that one could drop.

The only man who breathed this confidence during the Peloponnesian War besides Socrates was Aristophanes.

Aristophanes knew, in the way he appears to know everything, that in this time you could only breathe it in laughter, his kind of laughter which serves for reverence and respect. Alcibiades knew this confidence lives, but it always eluded his grasp although he traveled the world to seek it—when he knew perfectly well (but only Socrates could make him admit it) you could only find it at home.

This kind of unassuming confidence cannot be experienced without remembering Aeschylus, a man with strength enough to have compassion for a god. Significantly, during the time of the war it is only Aristophanes who could approach Prometheus with something equivalent, but at the same time entirely different from, the pitiless tenderness of Aeschylus.

Most of the spectators and judges at Socrates' trial knew nothing of this world of Athens before the Peloponnesian war except what they saw before them in Socrates. Plato was born ca. 429, Xenophon, who was not at the trial, perhaps in 435. Meletos, Socrates' official accuser, was perhaps Plato's age, certainly not much older; "a youthful defender of the youth," Socrates calls him. Ashamed at appearing in Court—for the first time in his life, he emphasizes—Socrates at his trial felt the weight of seventy years of living and the dignity they demand. He says he did not prepare a speech because it was not something for a man of his age to do—especially since his whole way of life with its love of justice speaks for him—in his defense.

Plato knew, of course, that Socrates came from another world; in fact one major part of his work was remembering and recreating a world he had never entirely known, but which he knew to be destroyed. Remember that Plato lost Socrates just after the experience of the Thirty had forced him to acknowledge the dishonor of his family—perhaps not of his parents, but of the brothers of one of his parents and of another close relative. His repudiation of their acts is strong, and it awakens admiration. For Plato, the trial of Socrates was a terrible as the time of the Thirty.

Plato's love for Socrates is for a dead man; everything he writes is about a man who has disappeared. Unlike Aristophanes, Plato never had to face Socrates with any of his writings. His writings were meant to substitute for Socrates, to replace him, to keep him alive once he was dead. This is the hardest illusion to deal with when you read Plato, the illusion that you are inside Socrates, that you are hearing his voice. It is also the drama of reading Plato, who is an artist, a different kind of artist than the poets, for he thought he was not an artist.

With Xenophon it was different. He stayed outside of Socrates. In Xenophon you can hear how Socrates' voice sounded to somebody who did not entirely understand Socrates but who knew he did not understand him, who knew he was out of his depth but had the courage to stay there—that is rare. "I cannot forgive him, I cannot forget him, the memories keep overwhelming me," Xenophon says somewhere with wonder. But unlike Plato, he never forgot they were memories.

Socrates was charged with impiety. The specific charge, which is preserved with slight variations by Xenophon and Diogenes Laertius, was that he did not worship the gods that the city worshipped and that he introduced new gods. The second charge is that he destroyed the youth. There are other examples of charges of *asebeia* [impiety] with other charges attached to them. For instance, Aspasia was charged with *asebeia* and letting Pericles meet free women in her house. There is a text of Aristotle that associates *asebeia* with disrespect for parents and corrupting the youth. In any case it is clear that corruption of the youth was a prosecutable offense.⁶

Plato's stress on corruption of the youth accords with Anytus's own views. In the only direct quotation from his speech we have, Anytus told the judges he had not expected Socrates to appear in court, but once he had, they had no choice but to condemn him. Otherwise, he would ruin their sons. In this Anytus agreed with the Thirty, who had actually attempted to order Socrates not to speak to the young,

Anytus's argument to the fathers to protect their sons is the strongest kind of appeal. As Socrates points out in his questioning of Meletus, it makes him responsible for all the troubled youths in the city. How lucky they would be if I am the only one who ruins them, Socrates remarks.

There is plenty of evidence of disturbed relations in Athens between fathers and sons during the Peloponnesian War. The son of Pericles in Xenophon speaks matter-of-factly of Athens as a place where sons held their fathers in contempt. You remember the struggle between Pheidippides and Strepsiades in the Clouds, where there is little question of the father holding the respect of his son. In the Birds there is a scene between a youth who desires to murder his father and Pisthetaerus, where Pisthetaerus manages to show him, by conversation not unlike those of Socrates in Xenophon, that he belongs on the Thracian front. Aristophanes means to show here-and it is probably meant as a compliment to Socrates-that the youth can be talked out of these wild fancies if there is anyone around who knows how to take the time to talk to him. (Incidentally, in our world, where we do not call things by their proper names, the would-be fatherkillers pass for revolutionaries.) There is in Xenophon also a remarkable conversation of Socrates with his son who is deeply angry with his mother. In all this we should keep in mind that disrespect for parents carried severe penalties, perhaps even death.7

These disturbed relations between fathers and sons were intensified by the war. Thucydides mentions the enthusiasm of the youth for the war at its beginning. Pheidippides would have been brought up in the country if it had not been for the war.

Socrates was one of the few people in Athens willing to look these troubles in the face rather than deny them and, by denying them, wish them away. Anytus instead wanted to wish them away, in somewhat the way Iocasta tried to talk Oedipus out of what he had learned—and then committed suicide. "Because I can help," Socrates says with something like astonishment, "I am overwhelmed by their jealous rage, as you put it, Euthyphro"—the word is *phroneo*, used elsewhere of the gods' resentment of overreaching human beings.

Anytus's relation to Meletus shows something of what Anytus thought the proper relation of the elder generation should be to the younger. He put Meletus up to charging Socrates, Meletus who was just a kid in Socrates' astonished but fearful eyes. How did he dare accuse him of impiety? Socrates asked. Did he not know what he was getting into? With a charge of impiety anything could happen.

Meletus was one of those young men for whom the world is unreal, for whom, as Socrates said of others, everything is upside down. He was one of those youths who wished to be serious but did not dare to be, who wanted to be a hero but feared the risks. Anytus offered him the easy way out, the illusion of self-respect, the easy way to grow up: the role of protector of the city and of his peers. Socrates is fierce when he questions Meletus, catching all the irresponsibility of that pretended earnestness. Anytus trapped Meletus with his conceit—and to all intents and purposes ruined him.

Contrast Anytus's manipulation of Meletos with Socrates's handling of Glaucon, Plato's brother, as Xenophon tells it. Like Plato, Glaucon at twenty wanted more than anything else to go into politics. Uncontrollable and the despair of his family, he was making a fool of himself climbing up and speaking in public, and doing the other things you did to have a political career at Athens.

Socrates cared about him because of Plato and because of Glaucon's uncle Charmides, and because he must have had all the charm of intelligence awakening. (There is always something important to be said for young men who dare to make fools of themselves in defiance of their family—as long as it is of their own—and not to please somebody else.)

Socrates asked Glaucon some questions which incidentally show something that I do not think is apparent from Plato, that Socrates had a fairly extensive knowledge of the facts of Athenian politics. He asked how long Athens could live off the agricultural production of the Attic countryside; how much food she needed in general; what her expenses were; what were her revenues—the list reads like a catalogue of the facts Pericles had at the ready when he spoke to the Athenians.

Glaucon could not answer any of these questions. At one point he answers, "But, Socrates, I can make a guess." "No, when you know, we will talk." Then Socrates asks him something else, "Why don't you run your uncle's estates?" Glaucon answers innocently, "Because I cannot persuade him to entrust them to me." "You cannot persuade your uncle, but you think you can persuade the city!"

This is pretty much the opposite of what Anytus did to Meletus. It is the kind of humiliating conversation which teaches the difference between dreams and facts, between illusion and life—without learning that distinction (and it is not something you learn in the head), you live your whole life among the shades.

Politics is also a struggle to distinguish the actual from illusions, enemies from friends, war from peace, what you can do from what you cannot, and, most importantly, aggression from goodwill and life from death. In the fifteen years preceding the trial of Socrates Athens had clearly failed in that struggle, over and over again misjudging the situations. When the consequences of those misjudgments turned to disaster, it grew difficult to put up with Socrates: he reminded people of too much. Without wanting to he made Anytus feel he was a bad father, and that there might be a connection between the kind of father he was and the kind of political leader he was. More generally, he made people feel they might have been responsible for what had happened to them. Or, as he puts it to the judges, "You cannot hurt me but you will hurt yourselves putting me to death."

Nobody in public life after Pericles, and probably not even Pericles, had been able to make people feel responsible for what had happened. Socrates made them feel responsible because he came in between the relations between generations. You remember how he says, "If I went abroad and had conversations, the fathers would drive me into exile; and if I did not, the sons would." In Athens it had taken collusion between generations, between Meletus and Anytus to prosecute him. For it is the relations between generations which determine whether cities live or die or merely survive.

People had gone through disaster; they had seen their fathers and brothers and children and friends killed. They had taken that, but they could not take the dim but unmistakable sense they had in the presence of Socrates that these disasters were of their own doing, that these disasters had to do with how they thought and talked and what they were. When Socrates told them they took better care of their slaves than their friends, of their bodies than their lives, he reminded them, quite unwittingly, of that.

Because he knew his own smallness Socrates struck other men as grand, boastful, even arrogant. Because he took his own measure, he appeared to tower over other men who had trouble telling themselves from gods. And this was intolerable, especially after the events of the last ten years had held up their smallness to them. A generation before they had laughed at him and respected him—now in the narrowness of defeat, possessed by memories they could not face, they killed him—because they feared themselves in him.

Notes

¹ For the relation of *The Clouds* to Socrates, Bruno Snell, "Das frühste Zeugnis über Sokrates," *Philologus* 97 (1948): 125-34; Wolfgang Schmid, "Das Sokratesbild der Wolken," *Philologus* 99 THE ST. JOHN'S REVIEW

(1948): 209-28; T. Gelzer, "Aristophanes und sein Sokrates," Museum Helveticum 13 (1956): 65-93.

² On the sources for this period, S. Accame, "Le fonti di Diodoro per la guerra Deceleica," *Rendiconti della R. Accademia dei Lincei* (Classe di scienze morali, sotriche e filologiche) 14 (sixth series), 1938, 347-451; "Trasibulo e i nuovi frammenti delle Elleniche di Ossirinco," *Rivista di filologia classica* 28 (1950): 30-49.

³ J. Hatzfeld, "Socrate au procès des Arginuses," *Revue des Etudes Anciennes* 42 (1940): 165.

⁴ For the character of this agreement, Ugo Enrico Paoli, *Studi sul processo Attico*, Padua, 1938, 121-142, especially 122; also, *Studi di diritto Attico*, Florence, 1930.

⁵ Burnet, *Plato's Euthyphro, Apology of Socrates and Crito*, (Oxford, 1970), especially v and the commentary to the *Apology*.

⁶ For the charge, A. Menzel, Untersuchungen zum Sokrates Processe, Abhandlung der Sitzungsber. d. kais. Akd. zu Wien 145, 1901-02, especially 7-29. Also E. Derenne, Les procès d'impiété intentés aux philosophes à Athènes au V^e et au IV^e siècles avant J.-C., Liége, 1930; N. Casini, "Il processo di Socrate," Iura 8, 1957, 101-120. For the association of asebeia with disrespect for parents, Aristotle "On Virtues and Vices," 1251a31:asebeia men he peri theous plễmmeleia kai peri daimonas e kai peri tous katoikhomenous kai peri goneis kai peri patrida. Also see J. Lipsius (with Meier and Schoemann) Das Attische Recht und Rechtsverfahren, Leipzig, 1908, 359.

⁷ For the death penalty for disrespect of parents (*kakõseõs goneõn graphē*), Lysias, 13, 91. As with many crimes, the penalty for disrespect of parents was probably not specified (for leeway in Attic legal procedure after conviction, U.E. Paoli, *Processo Attico*, Padua, 1938, 86-89. Recent scholars think disrespect for parents was usually punished with partial loss of citizen rights (*atimia*) for life, for instance, with loss of the right to hold office and speak in the assembly—but not with the loss of private rights, such as the right to hold property. L. Beauchet, *Histoire du droit privé de la République Athénienne*, Paris, 1897, 1, 362-371; A.R.W. Harrison, *The Law of Athens*, Oxford, 1968, 1, 78-81.



Publius Ovidius Naso relegatus non exsul. The Decree of Banishment

He envies the ocean, its coming and going,

its vast itinerary and easy arrogance.

Certain days he leans toward it, reaches

to touch, and touching touches what it has touched.

Then the poems come, torrential, unstoppable,

each riding the back of the one before,

their dolphin words like news from home.

Leonard Cochran

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Philosophy Revived Eva Brann

Those of our alumni who had really good Republic seminars won't have forgotten the spectacularly innocuous beginning of all philosophizing that Socrates sets out in the seventh book. He is talking to Plato's brother Glaucon about an education not so unlike ours. Where and how does reflection begin? With arithmetic. Take the one finger on your hand that doesn't even have an ordinary name, the fourth. It sends an odd message: It's always a finger, but it is long in respect to the pinkie, short compared to the middle finger. The attentive soul summons calculative thought to examine whether the eye's vision is announcing one or two things. If two, then each is other and one. And yet they are together a Twosome; is this Two itself a unity, a one? Now the soul is forced by its perplexity to inquire whatever One Itself might be. Counting and calculation is thus the "winch" to Being, and thoughtful arithmetic turns the soul around to that invisible Being. In another dialogue, the Phaedrus, quite another, an erotically ecstatic beginning of the ascent to Being will be set out.

Stewart Umphrey's book is the working out, meticulously reasoned and unflinchingly self-critical, of this humble finger problem and its relation to the second beginning in the ecstasy of love. But it is not a commentary on Platonic dialogues or on any of the writers—Hegel, Kant and a slew of moderns—that the author calls on. Though he puts to good use considerable philosophical learning, especially training in logic, it is an instrumental use. He wields the precision tools developed over two and a half millennia in the service of Socrates' great last care in the *Phaedo*: that the *logos*, thoughtful conversation, should live and not die with him. I used to think that commenting on major texts was by and large the most respectable kind of philosophizing, since it kept one honest, in the sense of preventing the inventive kind of originality that substitutes fascinating novelties for the same old truth. Stewart Umphrey's book, however, proves that there is honest originality—the kind that makes straight for the origin but by a unique route. It shows an independence of mind whose distinctiveness verges on eccentricity the kind that reaches the center by leaving the mainstream, whose perspective rouses thought by being a little askew, by shifting the world by a few degrees.

Stewart Umphrey's distinctive mode is a sort of faith-ful skepticism. His first book was in fact called Zetetic Skepticism, a title alluding to the hypothesis that searching inquiry (zetesis) can never be complete, that it is the indefeasible human condition not to be perfect in knowledge—and that this is absolutely no reason not to carry on; not despair but the energizing exercise of the intellect is the proper response. Complexity and Analysis is, among other things, a demonstration of the intellectual gymnastics that is at once a preparation for and an expression of this illusion-stripped faith. Of course, the reasons why it is best to keep inquiring are detailed along the way.

The Law of Contradiction with the divisions and branchings it induces is what this author lives by—to begin with, when the differentiations of analysis are wanted ("this is not that") and also later when distinctions in truth or falsity of statements about complex entities are to be made ("either this holds or that"). Some sections consist of very close reasoning, but the purpose is not to deconstruct polemically but to discover peacefully what is the case; it never gets irritating. At worst it is a sweaty workout, at best an illuminating high. Sometimes a long argument ends in misgivings after all; often the author, like a reasonable human being, admits to as-yetunreasoned preferences.

If readers may quail a little before the logic (yet—no pain, no gain), they will be enchanted with the distinctive flavor of the style, the underhanded wit, the anomalous vocabulary, the dry logic that supports as it contains the soaring of thought. Did you ever think of newsstands as an exemplification of "the truthvalueless things we make," or use "obvelation" as the obverse of revelation, or see a rational flow chart of philosophic madness? And then there is occasional lyricism. —If you thought you knew your tutor, read the last couple of pages of his book.

So far I've talked round about the book, but I want to give some notion of its contents. Since for Stewart Umphrey the good of philosophy is more (though not only) in the thinking than in the thought, in the doing rather than the having done, the text is practically beyond summary. Though it is full of passionate belief, it contains no thesis that won't be denatured by being baldly stated. So I'll be picking small samples, those that twang my logical funny bone or that speak to my philosophical preoccupations.

The book has three parts: The first is about analysis and its limits, thus about ways of knowing, epistemology. The second is about complexity, about the constitution of things that are discernible in thought (here called "entities"), thus about the inside workings of beings. (I have no idea why the order of the complementary parts is reversed in the title of the book, but have perfect faith that there is an interesting reason.) The last part draws the practical, the life-affecting consequences of the theoretical insights gained; it is thus about human wholeness and what lies beyond.

"Analysis" in this book is a way of trying to know some given whole by breaking it up into its constituents, be they themselves entities or something else, something central to the argument, namely "subentities," "parts" that are not independently countable or even discernible in analytic thought, yet somehow in evidence. Analysis deals comfortably only with fairly independent countable components. Analysts are clear and precise but not always deep and comprehensive. They can capture complexity but have trouble with unity. (They know that the finger contains two ideas, but not how Two comes to be one.) Some of the darkness of our day is due to a case of bad analysis, to "analysitis" ("Things fall apart; the centre cannot hold"), but reflectively used it is the beginning of understanding. Stewart Umphrey reviews the tensions that arise in mathematics and the natural sciences between analytic precision and comprehensiveness; the notes provide helpful sketches of several formalizations of this dilemma, two of which our seniors study: Gödel's incompleteness proof, showing that there is more in mathematics than proofs can reach, and the Einstein-Podolski-Rosen argument that the Heisenberg indeterminacy principle taken as a description of the physical world is incomplete. One principal aspect of what turns out to be the inherent paradox of analysis is thus the Socratic problem: If you analyze an entity into, say, two constituents, you've lost the one whole you were trying to understand.

In the second chapter of this part, Aristotle's metaphysics is considered as a model display of the insufficiency of analysis. This presentation not only advances the agenda of the book, it also enacts it: While showing why Aristotle has to become "transanalytic" it attempts to understand his several approaches (often made perspicuous in diagrams), and to comprehend in one understanding his notion of natural beings, of form and matter, essence and accidents, actuality and potentiality, and of divine being—unsuccessfully, of course. Anyone—all of us—who has ever wondered how these perspectives jibe will find this exposition independently illuminating.

"Complexity" describes the character of all thinkable and interesting entities. (Recall the trouble Plato's Parmenides runs into in trying to think simple unity, *the* One.) The types of entities are set out with respect to their constitution, from the extremes of mere pluralities or heaps to undifferentiated unities. In between are found the ones that matter: those with definite components that are nevertheless real unities and those that are real unities but with parts—subentities—not discernible in analysis. Entities are now inventoried; facts, for instance, which include relations or properties that seem to require a transanalytic "ontological glue" to hold them together with the entity itself. Living individuals are particularly complex entities, and with respect to them we can't help asking about their center or essence, what each being really is. But the familiar difficulty arises: The essence is what the individual is, but is it one and the same with or other than the individual? In the former case, knowing the essence adds nothing new to our understanding; in the latter it yields nothing true about the entity.

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Individuals take up or partake in space, and so space must be analyzed. Is it itself an entity, a whole? Are points its "parts"? They certainly belong to the understanding of space, but they are not its components. They are almost model subentities; space is thus a complex entity with constituents inseparable by analysis. The chapter on space is especially interesting, not just as an acute dissection of the spatial features, but because space is the field of display for the non-analytic thinking that is needed to understand the subentitive complexity which characterizes the wholes we most care about. This kind of thinking is analogical. Analysis should, at the least, give us clear and distinct components to go on with. But while clarity belongs to the content of an entity, to its own internal nature, distinctness pertains to its location, its externally delimited place in "conceptual space." Now the discernible components of a complex entity are particularly clearly and distinctly expressed in spatial representations (such as the author draws in the Aristotle chapter and throughout the book). But such spatial images also make visible indiscernible subentitive parts, for example points (see below).

Why is spatial representation so illuminating? Because we naturally analogize the mind itself to space—our spatial imagination is so close a neighbor to our intellect—and space is the most potent field there is. Moreover, being itself transanalytic, space is particularly apt for representing that kind of complex entity.

Individuals also partake in universals through their species. And universals in turn have similar problems: How is the species a part of the universal? Is the universal genus composed of species? Then where's the universal? So we again have Socrates' "One-out-of, or -over, or -beyond Many" perplexity. The author proposes yet another non-analytic kind of understanding which he calls dialectical prismatics: If we think of individuals (or species) as refractions of the universal, as white light is refracted into the color spectrum, then we might, by looking through the prism backwards, so to speak, sight the unitary source. Anyone trying to squint through appearances at what is thinkable in them will recognize the experience of reverse prismatics.

In the section on universals (part of the chapter on entities) we may, moreover, discover that this member of our community inclines toward "metaphysical realms of the *ante rem* sort," a fancy way of saying that ideas not only exist, but exist both separately and before the world. But that's Platonic, glory be!

Next God is viewed prismatically and then negatively. Like the reverse prism, the *via negativa*, seeing God through what he's not, is not analytic. The chapter on entities ends with a section called "Everything," for wisdom—which is what philosophy is for—is the comprehensive understanding of everything: whether all things form a whole and how, and if not, why not. Extreme analysts, such as the logician Quine, eliminate whatever it is that might make wholes of parts, such as universals. They slash away whatever isn't countable, as the ultimate comprehensive unity certainly can't be. They wield what is here wittily called "Quine's machete," the crude counterpart to "Ockham's razor," which deftly excises only the truly unnecessary. Extreme "haplists" (from Greek *haplous*, "simple"), believers in a simply single being, like Parmenides (though I don't know another), think that there

is one non-complex entity. Neither of their accounts is sufficiently accountable for what there is.

There follow three chapters on the ways of gaining understanding that can supplement analysis. The chapter on analogy, already mentioned, contains a sensible inquiry into "likeness," since analogy involves likeness; it is so much the more welcome because some recent estheticians have pretty much analyzed imitative resemblance away. The chapter ends with precepts for the safe use of analogies. The one I take most to heart is that philosophical analogies should be carefully framed as similes ("this is *like* that" rather than metaphor ("this *is* that"), and that some of the dead metaphors of our daily speech should be resurrected as instructive similes. (My favorite is the contrastive pair "*eidos*," that which is like something seen, and "concept," that which is like something grasped.)

Dialectic is a second transanalytic way. Dialectic is a term that has lived through many meanings. The author begins with Hegelian dialectic. Again, here, apart from this book's agenda, is a good account of the main moments of this developmental logic, whose chief feature is that all the analytic components are synthesized and reabsorbed into each other and the whole. It is complemented by a critique, an inventory of the "myths" underlying its beginning, its progression, its culmination, its very possibility for our thinking. A while ago, Stewart Umphrey, Chester Burke, and I together read our way—it took several years—through Hegel's *Science of Logic*, a great dialectical experience for all of us (which Stewart kindly acknowledges); this critique revived for me our perplexities of long ago—now neatly marshaled.

The author then turns to the dialectic best known here at the college: dialogue, conversation. "The soul of dialectic," he says, "is philosophy. Only secondarily are dialecticians elenctic [refutational] wizards." This kind of philosophical reasoning does not dwell in beginnings or completions but the in-between. It is both its own end (as a healthy activity) and for a purpose (as the search for wisdom). He says much more, of course, but the point is that dialectic implicitly affirms the philosophic condition of incompleteness.

Finally there are the ways of negation. "Is it possible to combine the metaphysical splendor of infinitistic realism [faith in vague beings] with the epistemological refinement of finitistic irrealism [unbelief in any imprecisely delimited entity]?" asks the author. "No," he answers. "We're caught then between two opposing views, one of which appears decidedly pinched, the other decidedly vain." Then follows a critique of Kant's transcendental dialectic, insofar as it means to show that our deepest philosophic desires must remain forever unmet because the dilemmas we burden ourselves with are based on an illusion-that we can know things in themselves apart from the constructions of our understanding. The author shows precisely something that is eventually divined by every student of Kant: the Critique of Pure Reason is perforce shot through with the very transcendence Kant wants to cure. It is a book far more self-refuting and thus far more suggestive than the first laborious reading reveals.

Kantian naysaying having been analyzed, a praise of wonder and perplexity (carefully distinguished, of course) is mounted in a final section on intuition. Here the impersonal "one" gives way for some reason to the politically correct "she"—an object lesson in the trickiness of that business, since the attribution of intuition to women is, as it happens, quite politically incorrect. I had to grin.

The third part consists of three enticing chapters, Integrity, Ecstasy and Community: how human beings become whole and how they transcend themselves as persons and also as social beings.

The chapter on integrity culminates in a section about "presence of mind," a kind of undefined summary virtue capping the qualities of "reason, choice, character," to which are opposed "will, commitment, appetite, freedom." I find Stewart Umphrey's preference for unity and integrity over plurality and disorder deeply agreeable; others may find it usefully provocative. In any case presence of mind is the oldest intellectual virtue known to the West: wise Athena once calls her darling Odysseus *angchinoos*, "present-of-mind."

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The chapter on ecstasy presents a sober analysis of ways of going beyond oneself, especially in love; its conclusion presents the union of integrity (wholeness) with transcendence (going beyond) in the "sober madness of philosophy."

What is remarkable is that the careful descriptions and distinctions of these human chapters really are applications. They are the practical consequences of the preceding pure theoretical thinking when brought to bear on living experience; they trace out incompleteness as a human condition.

The final chapter, on community, begins with the political, the "most authoritative," human community. Within it we work ourselves into integrity, out of it we pass beyond ourselves. Careful consideration shows that although it is the basic ground of our well-being, it is imperfect as a community. Friendship appears to be the realization of a paradigmatic community. So an analysis of friendship is in order: "Friendship is complex. It involves a relation that is usually dyadic and always symmetrical"—so this ultimate topic is broached with the formal precision and logical perspicuity of the inveterate analyst who relishes his human reserve; I have to smile even as I'm thinking along. But the logic eventually yields to love, and, as I've said, the last two pages of this book speak of the most intimate of human experiences.