The Probability Function in Quantum Mechanics:

A Formal Cause Beyond Space and Time

Sihui Ma
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In our ordinary lives, we tend to persuade ourselves of the existence of an object or an event by its occurrence in space and time. When we speak about an object or an event, the first questions we wish to ask are "where is it?" or "when did it happen?". An accurate description in space and time has always been the criterion for truth and reality. The science of quantum mechanics, however, has led us into an exotic territory where our intuitions of space and time become unreliable. Instead, we encounter strange concepts such as Planck's constant, *h*, and the probability wave. Does the science of quantum mechanics suggest that there exist irremovable impediments on our path that stop us from revealing the secrets of nature? Or, could these new and counter-intuitive concepts replace space and time in shedding light on our understanding of nature and reality? To inquire into Planck's constant and the probability wave function and their interconnection, let us leave for now the world with which we are familiar and venture into the realm of quantum mechanics.

I. The discovery of Planck's constant, *h*

> *Many people do not understand the sorts of thing they encounter! Nor do they recognize them even after they have had experience of them—though they themselves think they recognize them.*[1]
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> *—Heraclitus*

The mysterious *h* makes its debut in thermodynamics and is essential to Max Planck's theory of heat radiation, which saves the phenomena of black-body radiation from the "ultraviolet catastrophe" implied by the Rayleigh-Jeans formula. In the classical wave theory,

1. Clement, *Stromateis* 2.8.1: οὐ γὰρ φρονέουσι τοιαῦτα (οἱ) πολλοί, ὁκόσοι ἐγκυρεῦσιν, οὐδὲ μαθόντες γινώσκουσιν, ἑωυτοῖσι δὲ δοκέουσι.

when a black body is heated in an oven, it absorbs heat energy and then emits electromagnetic waves with all possible frequencies. Rayleigh and Jeans calculate their formula for the energy density by using the classical law of equipartition of energy, which states the energy is partitioned equally over all frequencies.[2] Their theory is that the energy density increases significantly as the frequency increases. Their prediction, however, matches the experimental results only at low frequencies, while the empirical data show that at very high frequencies, such as in the ultraviolet region, the energy density goes to zero instead of approaching infinity, as their formula predicts.

Meanwhile, Planck seeks a way to alter the Rayleigh-Jean equation such that it could accurately predict the experimental results at all frequencies. He does one trial on the Law of Equipartition, $\varepsilon = k \cdot T$, in the Rayleigh-Jean formula, which he replaces by $\varepsilon = \frac{h\nu}{e^{h\nu/kT}-1}$. So far, h does not carry any inherent meaning. It is just part of Planck's guess, but it is a lucky guess: The presence of h in the equation allows the revised theory to correspond to the empirical data. Not satisfied with an ad hoc formula, Planck proceeds next to determine the physical meaning of h. He interprets h as a proportionality constant that helps predict the amount of energy distributed to electromagnetic waves with different frequencies. As an electromagnetic wave oscillates with a certain frequency ν, it will not emit energy continuously but release integral number of units $\Delta\varepsilon = h\nu$, which Planck calls "energy quanta". Planck's idea of quantized energy has posed considerable challenges to the concept of continuity that lies as the foundation of classical Maxwellian wave theory, in which energy is a continuum that can be subdivided ad infinitum

2. The energy density of a cavity is the average energy content per unit volume of the cavity.

and is absorbed and emitted continuously by bodies. Planck, nevertheless, does not intend to question the concept of wave and the continuity of space. His law demonstrates that energy is only absorbed and emitted as discrete packets, but energy does not travel in space in the form of particles. The electromagnetic waves are not discarded by Planck: They still serve as the carriers of energy in space as a continuum. Planck attempts to attribute both continuity and discreteness to energy: It is discrete when entering or exiting a hot body, but continuous when traveling in space. The presence of h has not yet threatened the continuity of space and time, inasmuch as discrete energy must still propagate as waves with particular frequencies in space and time which themselves are continuous.

While the existence of h leads Planck to partially deny the continuity of energy (energy is discrete only when being absorbed by or emitted from a hot body) without infringing the wave theory, Albert Einstein, in his paper on the photoelectric effect, takes one step further to question the adequacy of the wave picture itself. Before exploring Einstein's view about electromagnetic waves, let us first examine some facts about the photoelectric effect. Philipp Lenard shows that "[W]hen a photosensitive metal was made the cathode in a cathode ray tube and then illuminated, a measurable photoelectric current, carried by these emitted electrons, passed from cathode to anode."[3] In experimenting on the photoelectric effect, Lenard discovers two striking physical correlations between the properties of the light source and those of the electric current. First, increasing the intensity of the light source will proportionally increase the current from the cathode ray, i.e. the number of the released electrons. Second, the kinetic energy of each released

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3. *Senior Laboratory: Atoms and Measurement* (Annapolis: St. John's College, printed in 2016), 97.

electron is proportional to the frequency of the illuminating light.

Based on James Clerk Maxwell's electromagnetic wave theory, an increase in light intensity is equivalent to an increase in the intensity of electromagnetic field of the light wave, which should instantaneously increase the electromagnetic force acting on the electrons and thus increase their speed. This assumption, however, is contradicted by the photoelectric effect in which the speed of the electrons is not correlated with the intensity of the illuminating light but with its frequency. Einstein describes this correlation mathematically as $E = h\nu$, where E is the energy of each individual electron, and ν is the frequency of the illuminating light. Einstein's "heuristic point of view" about photoelectric effect has several significant consequences.[4] He connects the concept of h to quantized energy by describing h as energy divided by frequency. In classical physics, frequency is a discrete physical quantity: It represents the number of oscillations or beats within a certain amount of time, i.e. the period, and therefore frequency must increase and decrease discontinuously. In this context, h indicates the amount of energy generated by each oscillation. Since ν changes discontinuously and h is a constant, the total energy of the electron E will also change discontinuously. Einstein has thus expanded the idea of energy quanta from the field of thermodynamics to electromagnetics. The constant h is no longer just a lucky guess but essential to the idea of quantized energy: It is understood by Einstein as the smallest unit of energy.

Einstein, however, is not completely satisfied with Planck's assumption that energy is

4. Albert Einstein, "Concerning a Heuristic Point of View about the Creation and Transformation of Light", in *Annalen der Physik*, 17, trans. Editors of *Annalen der Physik* (1905), 132-148.

only absorbed and emitted as units of h. He further assumes that energy in light is also distributed discontinuously in space:

> *According to the presently proposed assumption, the energy in a beam of light emanating from a point source is not distributed continuously over larger and larger volumes of space but consists of a finite number of energy quanta, localized at points of space, which move without subdividing and which are absorbed and emitted only as units.*[5]

Here Einstein starts to define the idea of energy quanta with respect to space. We can interpret this connection between h and space in three ways. First, the theory of discrete energy quanta travelling in space entails the particle picture. Each quantum of energy has a "shape" or form in space which distinguishes itself from other quanta. While in classical physics the notion of particle only applies to material substance such as a hydrogen atom, Lewis Gilbert later decides to name the energy quanta of light that Planck and Einstein discover as "photons", though they do not possess mass: "I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an essential part in every process of radiation, the name photon."[6] The concept of particle seems to encompass both energy and matter, and the distinction between energy and matter diminishes in significance. Second, Einstein's conceptualization of light quanta as particles foreshadows the duality of wave and particle. Light is indeed quantized in the photoelectric effect. The frequency v in Einstein's equation does not pose a challenge to the particle picture, since frequency is a property that describes both the

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5. Ibid.

6. Lewis Gilbert, "The Conservation of Photons", in the *Letters to the Editors*, (Berkeley 1926).

oscillation of a wave and the vibration of a particle. There exist, nonetheless, other optical phenomena that can only be explained by the wave theory, such as the refraction of light and Young's double-slit interference. The conflict between the wave and the particle pictures of energy begins to take its shape. Third, there is one view shared by both Planck and Einstein: The continuity of space. Since Einstein claims that light travels as particles in space, the space needs to be continuous for the word "travel" to carry a real meaning. The concept of motion implies continuity, for if there is a gap in space an object will no longer move or travel, but will take leaps instead. Hence, *h* has not yet intruded into the Newtonian concept of an absolute and continuous space. Energy is discrete, but space is still continuous.

II. The conflict between *h* and space-time continuity

> *One must realize that war is common, and justice strife, and that all things come to be through strife and are so ordained.*[7]
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> *—Heraclitus*

The conflict between the constant *h* and the continuity of space-time downplayed by Planck and Einstein comes to the forefront when Niels Bohr attempts to devise a theoretical explanation for the pattern of spectral lines of the hydrogen atom. In his paper *On the Spectrum of Hydrogen*, Bohr claims that the phenomena of the spectral lines reveal the inadequacy of Rutherford's atomic model based on classical electromagnetic theory. In Rutherford's atomic model, the nucleus, which carries all the positive charges and whose size is negligible compared

7. Origen, *Contra Celsum* 6.42: εἰδέναι δὲ χρὴ τὸν πόλεμον ἐόντα ξυνόν, καὶ δίκην ἔριν, καὶ γινόμενα πάντα κατ' ἔριν καὶ χρεών.

to the size of the atom, is situated at the center of the atom. The electrons each carry a unit negative charge, the sum of which equals the positive charges at the nucleus, and revolve around the nucleus in elliptical orbits. Rutherford's ideas about his atomic model can be traced back to Sir Issac Newton, for this atom is described not unlike a universe in a nutshell: The electrons are like little planets which orbit around the nucleus, the atomic star. Bohr, nevertheless, has disproved this mini-universe theory by examining its consequences from an understanding of spectral radiation.

How is the atomic model related to the phenomena of spectral lines? In the classical electrodynamic theory, an electron which orbits around the nucleus emits an electromagnetic wave whose frequency equals its own frequency of revolution. Bohr expresses both the frequency of revolution ω of the electron and the diameter of its orbit *2a* in relation to *W*: $\omega^2 = \frac{2W^3}{\pi^2 e^4 m}$, $2a = \frac{e^2}{W}$.[8] Since energy is continuous under the assumption of classical wave theory, W may take on all possible positive values, which means that the changes in both *ω* and *2a* are continuous. An electron in the atom can thus take on all possible orbits and emit electromagnetic waves with all possible values of frequencies. If this hypothesis is true, then we should detect a continuous spectrum of light when observing the spectral lines of hydrogen in a Geissler tube. On the contrary, what we observe in the tube is not a continuum but always a series of broken lines. The individual lines suggest that only certain frequencies of the electromagnetic waves are emitted by the electron. If the frequency *ω* does not have all possible values, then according to

8. m and e are the mass and the charge of the electron, while W is the work which must be added to the system in order to remove the electron to an infinite distance from the nucleus.

Bohr's equation *W* and *2a* cannot change continuously because they could only take on certain values. Bohr has thus arrived at an absurd conclusion: The electron in the hydrogen atom can only take certain orbits and obtain certain values of energy. As the electron "jumps" from one orbit to the adjacent one, it loses or gains a certain amount of energy by emitting or absorbing lights with fixed values of frequencies. Bohr defines this relationship between increments of energy and the frequency of the spectral lines by borrowing Einstein's equation: $\nu = \frac{E_1}{h} - \frac{E_2}{h}$.[9] Hence, the spectral lines of a hydrogen atom are manifestations of the gaps between two possible orbits of the electron. We can also conclude from the equation that h is the cause of such gap, for when h approaches 0 the numerical difference between the two energy states vanishes and they become a continuum.

At this stage, at stake is the concept of continuity, which has been severely challenged by the existence of h: Not only is energy discontinuous, but space itself might also have gaps. A difficult question arises from Bohr's theory of the "quantum leap": When the electron changes its orbit, what happens between the two adjacent orbits? There are two possibilities based on whether we regard the space as a continuous or discontinuous physical magnitude. If we preserve the continuity of space, then the electron must traverse the space between two orbits instantaneously, which requires the electron to possess infinite speed. This conclusion cannot be valid, since according to Einstein's theory of special relativity nothing can travel faster than light, the speed of which is large but finite. Or, if there is no space at all between the two orbits, the

9. E_1 and E_2 are the two energy states of the electron.

electron does not "move" in space but instead "leaps" from one orbit to another. A parallel situation would be that a person chooses to walk down a hill through discrete descending staircases instead of traversing a continuous downward slope. The person in this the analogy is, however, crossing over the space between two adjacent staircases, but what is the electron leaping over if there is no space between the two orbits at all? In either possibility, we cannot describe a path of an electron in the hydrogen atom, and the concept of motion can no longer be applied to describe the elementary particles. That is why Bohr writes "we stand here almost entirely on virgin ground."[10] For the reason that the classical theory cannot reconcile the existence of h with the continuity of space, physicists like Bohr are craving for a new scheme which explains quantum phenomena with a clear picture in space and time.

As we can see from Bohr's atomic model, the presence of discrete energy quanta h leads to the breakdown of space-time continuity. One way to save the continuity of space-time, however, is to understand everything as wave instead of particle. Since the particle theory of energy has already been well established by both Planck and Einstein, scientists endeavor to combine the wave theory with the particle theory. Any physical entity, whether matter or energy, can exhibit the properties of both a wave and a particle. Since matter is understood as particles in Newtonian Mechanics and energy is understood as waves in Maxwellian electrodynamics and classical thermodynamics, the key to developing the wave-particle duality is to unite the concepts of matter and energy. We have already seen the amalgamation of these two concepts in

10. Niels Bohr, "On the Spectrum of Hydrogen", in *The Theory of Spectra and Atomic Constitution*, (Cambridge, 1922).

Einstein's view of the photoelectric effect, which characterizes the discrete energy quanta of light as particles. His understanding has opened up the possibility of the other side: If energy could be particles, then matter could be waves as well.

De Broglie postulates the concept of "matter-wave" which complements the concept of photon as an effort to unify the "Physics of matter and the Physics of radiation."[11] He assumes that for each material body there corresponds a particular matter wave, and he derives the mathematical relation between a body and its matter wave by combing the Maxwellian wave theory of light ($p = \frac{E}{c}$) and Planck-Einstein's quantum theory of light ($E = h\nu$): $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$.[12] Since in De Broglie's formula h is the product of the momentum of the particle and the wavelength of its corresponding matter wave, h becomes the key to linking the concept of wave to particle. De Broglie's theory of matter wave presents a plausible explanation for the "quantum leap" in Bohr's atomic model. If we understand the revolving electrons not as particles but as matter waves, then an orbit is formed when its circumference is an integral multiple of the wave length of the matter wave, for only under this condition can a standing wave be formed.

Another problem, however, arises due to the characteristics of the matter wave. This wave differs fundamentally from the electromagnetic wave in Maxwell's theory in that the matter wave cannot be a physical wave. De Broglie assumes a relationship between the speed of

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11. Prince Louis V. de Broglie, "The Undulatory Aspects of the Electron", in *The world of the Atom*, Vol. II (New York: Basic Books, 1966).

12. P and E are the momentum and total energy of the material body, c is the speed of light, and λ is the wave length of the matter wave.

the particle w and that of its corresponding matter wave u: $u \cdot w = c^2$.[13] This equation entails something bizarre: Either the speed of the particle or the speed of the matter wave exceeds the speed of light, or both are equal to the speed of light (this last case applies only to photons rather than to any other material bodies). Any of these three cases would violate Einstein's theory of special relativity, based on which nothing can travel faster than light. De Broglie thus comes to postulate the matter wave as a "guiding wave" of the particle, which keeps the particle traveling in phase with the guiding wave, even though they have different speeds. The guiding wave provides the particle with a hypothetical path to travel, but the wave itself is not in space. The matter wave cannot be a physical wave but only an imaginary mathematical wave, for if the matter wave is physical the wave-particle picture will contradict the theory of special relativity. For the sake of preserving the continuity of space and time by his theory of wave-particle duality, De Broglie is forced to take his matter wave out of space.

Schrödinger, nevertheless, is not fond of De Broglie's notion of the "guiding wave". Schrödinger wants a real physical wave as an explanation for the behavior of each particle. To avoid the difficulty of picturing the De Broglie guiding wave, Schrödinger tips the balance of the wave-particle equilibrium by adopting the wave as the ultimate physical reality. A physical object cannot both be a particle and wave in space and time: All physical entities which we perceive as particles are in fact composed of waves with different frequencies. Schrödinger replaces the physical concept of particle by a wave packet. When a group of waves with various frequencies are superimposed on each other, they form one wave as the result of interference. There will be

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13. c is the speed of light in empty space.

constructive interference within a particular region, outside of which they cancel each other's amplitude through destructive interference. The portion where the waves strengthen each other thus forms a wave packet, and this wave packet can move in space as its component waves propagate through space.

According to Schrödinger, we can determine the wave which corresponds to a particle through his equation: $\left\{H\left(\frac{h}{2\pi i}\cdot\frac{\partial}{\partial q},\ q\right)+\frac{h}{2\pi i}\cdot\frac{\partial}{\delta t}\right\}\Psi=0$ or in a simplified form: $\left\{H(p,\ q)-E\right\}\Psi=0$.[14] The Schrödinger equation informs us that once we know the position and momentum of a particle and its total energy, we could solve for Ψ which is the wave that corresponds to the particle. The momentum of the particle is proportional to the partial derivative of the Ψ function with respect to its position, while the energy of the particle is proportional to the partial derivative of the same Ψ function with respect to time. It is striking that both the differential equations for momentum and energy share the same coefficient $\frac{h}{2\pi}$. This shows that Schrödinger incorporates the discrete unit *h* into his wave function which is itself continuous. Now the discrete particles can be described as continuous waves. Schrödinger aims to disprove the discontinuity of space and time by attacking its cause: The particle theory. The discreteness is nothing but an approximation, an ambiguous concept, and a composite effect of a group of continuous matter waves with different frequencies. The *h* seems to have been brought to peace with space-time continuity through the Schrödinger equation.

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14. H is an energy operator of the Hamiltonian theory and is a function of position q and momentum p: $H(p,q)=\frac{1}{2m}p^2+V(q)$, where V(q) is the potential energy of the particle.

III. The Indeterminacy Principle: a sharp divide between two worlds

> *They are separated from that with which they are in the most continuous contact.*[15]
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> *—Heraclitus*

Schrödinger's effort to reunite wave with particle, nonetheless, is frustrated by Werner Heisenberg's principle of indeterminacy. Heisenberg points out a problem about this wave packet hypothesis based on Schrödinger's equation. Suppose multiple waves form a wave packet within the region of Δx and cancel out each other outside. As the wave packet with length Δx travels in space, not only does it move forward but it also spreads and disperses itself in all other directions.[16] Once the packet disperses in space, it loses the characteristics of the particle, and thus the integrity of a particle cannot be preserved when it is hypothesized to travel in space as a wave packet.

A mathematical formulation of the effect of this dispersion is Heisenberg's principle of indeterminacy. The dispersion of the wave packet Δx results in the change of its velocity, since the velocity of the packet is determined by the composite velocity of the individual component waves. We cannot predict with certainty how this velocity fluctuates, just as we do not know how the wave packet disperses itself in space. While momentum equals mass times velocity, an indeterminacy in velocity necessarily leads to the indeterminacy of the momentum of the wave

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15. Marcus Aurelius 4.46: ᾧ μάλιστα διηνεκῶς ὁμιλοῦσι λόγῳ τῷ τὰ ὅλα διοικοῦντι, τούτῳ διαφέρονται.

16. Werner Heisenberg, "Critique of the Physical Concepts of the Particle Picture", in *The Physical Principles of the Quantum Theory*, trans. C. Eckart and F.C. Hoyt, (Chicago, 1930).

packet Δp. Heisenberg defines this degree of uncertainty in relation to h: $\Delta x \cdot \Delta p \geq h$.[17]

We can also interpret the principle of indeterminacy in terms of the interactions between an observer and the observed system. There are two possible situations in which we observe an object. First, our act of observation does not change the properties of the object under observation. This applies to all observations in the Newtonian world, where there is no indeterminacy. When we open our eyes and see a tree, our seeing activity does not change the tree, its position, its shape or its color. The tree exists in space, whether we are observing it or not. The other scenario is that in observing a system we do cause an irreversible effect on the system, an effect which, as Heisenberg believes, comes from the measurement of an electron or in general any determination of a quantum system. When we determine the position of the electron, the photon which strikes our eyes and enables us to detect the electron has already changed the momentum of the electron under observation through collision. In other words, in the Newtonian world h is regarded as 0 or infinitely approaching 0, while in the quantum realm the quantity of h can no longer be ignored. h stands in the principle of indeterminacy as a limit between the two worlds, one that allows a space-time picture of a system and the other that does not.

Heisenberg believes that the realities of both worlds can be preserved, as long as we are careful with distinguishing events at macro scale from micro scale and describing them with different categories of physical concepts. The conflict between classical physics and quantum physics arises as soon as we abuse physical notions which only apply to objects at the macro level by attributing them to subatomic particles. It is the core of the principle of indeterminacy to

17. Ibid.

highlight the problem of misusing concepts and to mark a definitive boundary between the two scales. The principle of indeterminacy informs us that both the position and momentum of an elementary particle cannot be accurately determined simultaneously. The range of both its position and momentum, when multiplied, has the value of Planck's constant as its lower limit.

Heisenberg thinks there are two ways to interpret the indeterminacy relations with respect to the act of measurement. The first account is that the crudeness of our measuring devices causes the value of position and momentum to deviate from the accurate result. Different methods of observation or measurement, as Heisenberg recognizes, do influence the accuracy of data we procure in experiments. The inaccuracy resulting from our own measuring errors, however, is not the only kind that the principle of indeterminacy addresses. If so, the indeterminacy relations would not make any qualitative distinction between the Newtonian world and the quantum world, because the errors caused by the crude apparatus could occur in both realms. On the other hand, such inaccuracy of measurement is not insurmountable: If we refine our measuring devices or improve the methods of our experiment, such errors can be eliminated in theory. To put it differently, if our own method of measurement is the only cause of the indeterminacy relations, we may hope to reduce the right-hand side of inequality from h to 0. Had we been able to determine the position and momentum of a particle with exactitude, that is to reduce the indeterminant degree of our measurement below h by eliminating our measuring errors, there would be no problem of misapplying the classical concepts to quantum phenomena.

The second account is that the degree of indeterminacy marked by the letter h exists not due to our subjective errors in measurement, but exists objectively as a property of an elementary particle itself. This indeterminacy manifests itself as the deficiency of our knowledge of position and momentum whenever a measurement or determination takes place. The problem of indeterminacy at the quantum level arises not from how we measure, but the fact THAT we measure. At the quantum level, each act of measurement will cause a considerable disturbance on the system under observation. Why does our measurement affect the state of a subatomic particle far more considerably than that of a Newtonian object such as the moon? Let us explore this question by comparing the different meanings conveyed by the concepts of position and momentum in classical physics and quantum mechanics. In the macro world, we can simultaneously measure the position and momentum of the moon with exactitude (provided that we have a telescope with very high resolving power) and therefore predict its future course with certainty according to laws in Newtonian physics. This simultaneous determination of both properties of the moon rests on the assumption that the moon is in no way affected by our looking through the telescope. We take this assumption for granted, because the positon and momentum of the moon we measure are mean values of the positions and momenta of all the elementary particles which comprise the moon as one massive object. The light which is reflected from the surface of the moon to our telescope may have altered the momentum of one or several electrons on the moon, but the effect the stream of light has on the momentum of the moon as a mean value of momenta of all its component particles is negligible.

In contrast, if we are to determine the position or the momentum of one individual electron, either our knowledge of its exact position or momentum or both must be sacrificed based on the restrictions of indeterminacy relations. The influence arising from our observation of one quantity is considerable enough to change the other, since in this case we are no longer measuring the position and momentum as mean values of a macro object as one system. Hence, the uncertainty of a particle's position and momentum can also be interpreted as the confusion of concepts in classical physics which happens when we carry these concepts over to quantum physics.

There are two solutions to this misuse of physical concepts. Either, the ordinary concepts we use in classical physics such as position and momentum must be forfeited and we must invent a whole new system of concepts in order to render accurate descriptions of events at the quantum scale. Or, we can compromise by preserving these concepts, but placing their applications under certain constraints. Heisenberg proposes the second solution: Taking into consideration the limit of accuracy prescribed by the principle of indeterminacy allows us to preserve the meanings of position and momentum at the subatomic level. This compromise seems more advantageous than the total abandonment of classical concepts, insofar as we can still describe the events at the subatomic level under the frame of space-time, albeit not with accuracy. A cloud chamber, for instance, can inform us of the approximate "path" of an electron, if we allow a certain degree of variance of its position and momentum at each moment, yet the image of the path of the electron will be blurry due to the indeterminacy relations. The value of Planck's constant in the expression of indeterminacy plays an important role in determining when the image of a moving object becomes blurry, by marking the proper boundary between the macro level and quantum level. In the Newtonian world where the value of h is considered vanishingly small, we can have a clear space-time picture of a moving object because we can measure both its position and momentum simultaneously and predict its motion in the future. When the object we observe shrinks to the size at which the value of h can no longer be ignored, such as that of a photon, the image becomes blurred as a result of our inability to acquire simultaneous knowledge of both its position and momentum. The value of h, therefore, specifies the highest resolution of the image of an elementary particle we can have.

IV. The probability wave vs. Aristotelian potentiality

A road up and down is one and the same.[18]

—Heraclitus

One of the fundamental disparities between the Newtonian world and the quantum world is that in the latter our observation cannot be continuous. We can observe the continuous motion of a metal ball rolling from the top to the bottom of a slope as in Galileo's experiment. The metal ball, like all objects in Newtonian world, possesses a definite path in space and time regardless of our observation. In contrast, if we detect an electron first at point A and then at point B, due to the indeterminacy principle there is no way we can know what occurs between the two measurements because our act of observation disturbs the electron. Heisenberg remarks: "We cannot describe what 'happens' between this observation and the next."[19] The underlying reason is not that the electron took a path from point A to B and escaped our notice. For if this interpretation were true, we would then be back in the Newtonian world where every object has a

18. Hippolytus, *Refutation of All Heresies* 9.10.4: ὁδὸς ἄνω κάτω μία καὶ ὡυτή.

19. Werner Heisenberg, "The Copenhagen Interpretation of Quantum Theory", in *Physik und Philosophie*, trans. C. Burke and E. Brann, (Stuttgart, 1959).

definite path. Heisenberg believes that the electron has no path between A and B: Whatever occurs between the two observation does not allow for a space-time picture. Max Born, whom Heisenberg joins in framing the Copenhagen interpretation, proposes the probability interpretation: An elementary particle such as an electron and a photon exists as a probability wave before our measurement, which is described by the Schrödinger's wave equation. Only when we start measuring the electron does it turns into a particle in space.

What is a probability function and what is the distinction between the probability state and the observed state of a quantum system? Let us explore these questions together with Heisenberg's comments on Aristotle's dichotomy of potentiality and actuality throughout his discussion on the probability state in *Physics and Philosophy*. As he attempts to illustrate the distinction between the two states of a quantum system, the one before and the one after our measurement, Heisenberg refers to Aristotle's account of potentiality and actuality as an analogy. Heisenberg's assumption is that an act of measurement during one scientific experiment is identical to an act of seeing in the Aristotelian sense. In *De Anima*, Aristotle characterizes seeing as an activity which actualizes the object one sees: "The activity of the sensible object and that of the percipient sense is one and the same activity."[20] Aristotle's definition of reality fits better into quantum events than does reality in the Newtonian sense. In Newtonian world, reality is entirely objective and irrelevant to our observation. We can confirm the reality of a tree by seeing it, but our act of seeing has no bearing on its existence. It does not matter whether we stare at the tree, close our eyes, or even turn our back against it. The tree remains at the same place in all

20. Aristotle, *De Anima*, trans. J. A. Smith, (New York: Random House, 1941), 425b 27-28.

three situations: It neither disappears nor moves to another place. Aristotle, however, attributes both subjectivity and objectivity to reality. Before we see the tree through the light, the tree exists, but only as potentiality. It is only through our interaction with the tree, namely seeing it, that the tree comes to be as actuality. As soon as we close our eyes, the tree sinks again into the status of potentiality, due to the absence of our subjective interaction with the object.

Likewise, the existence of a subatomic particle, such as an electron, also depends on our interaction with the quantum system. Let us perform a thought experiment, in which electrons are released from a cathode ray directly facing a piece of fluorescent screen. Based on Born's probability interpretation of Schrödinger's equation, an electron exists as a wave before we put a screen against it. In opposition to Schrödinger, who thinks that elementary particles exist all the time as physical waves like water waves or sound waves, supporters of the probability interpretation claim that this wave is not a physical wave, but a probability wave. This means we cannot locate this probability wave in space other than become informed by it the tendency of the electron, which is not yet in space, to be in different places. It is only after our measurement takes place (when the fluorescent screen is illuminated), that an electron comes to be in space.

To further elaborate the difference between treating the wave Ψ as a physical wave and as a visual representation of probability, let us imagine a hypothetical situation in which an electron is emitted from a cathode and travels in free space without obstacle. Let Ψ be the function which expresses the Schrodinger wave associated with the electron. Suppose we take a snapshot of the probability wave at time t_1 and represent its location as in Figure I, and then choose an arbitrary point A (x_1, y_1, z_1) on the wave function of the electron Ψ. According to Schrödinger himself, the value of the function Ψ_1 at this point informs us of the amplitude of the physical wave, or the electric charge that it carries at a location in space (x_1, y_1, z_1) at a time t_1. The electric charge carried by the wave exists at that location. For Born, Ψ_1 does not tell us an electron exists at the location, but how likely it is that if we took a measurement, we would find an electron there. Suppose we keep the time t_1 constant and find another point B (x_2, y_2, z_2) on the graph such that $\Psi_2 > \Psi_1$, then the probability of finding the electron at point B, P_2, is greater than the probability at point A, P_1 at time t_1. The electron at time t_1 does not yet exist in space, though it has the potential to exist at point A or point B or in any region that the probability wave occupies.

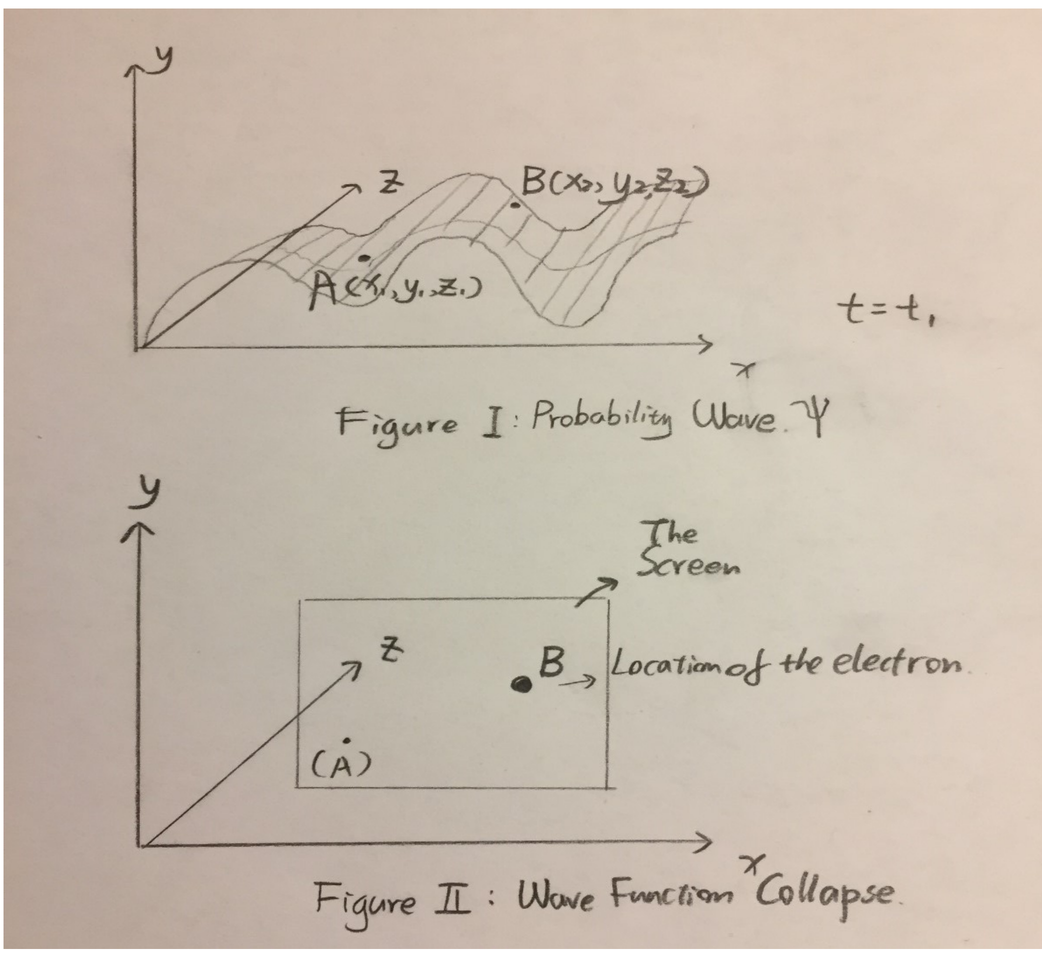

Figure I: Probability Wave Ψ

Figure II: Wave Function Collapse

Now we place a fluorescent screen in front of the cathode as shown in Figure II, so that eventually the electron will collide with the screen. Assume that at time t_1, we observe that the electron hits the screen at point B (x_2, y_2, z_2) instead of point A. As the result of our measurement, the probability P_2 of finding the electron at point B_2 becomes 1, while P_1 of point A and the probabilities of detecting the electron at any other point on the screen shrinks to 0. This means the wave Ψ now only has one value at point B, while its amplitude at all other points on the graph becomes 0. In other words, our act of measurement forces the probability wave to collapse into one point (or a packet, if we take the volume of the electron into consideration).

In his analogy, Heisenberg considers a space-time picture as essential to the actuality of a quantum system. This is also why he is equating an act of measurement to an act of seeing. When we see an object, we immediately receive information about its location at one particular moment. A measurement of a quantum system plays a similar role as our seeing activity: Even though we cannot see the electron directly, a detection of a tangible effect of the electron, say a bright spot on the fluorescent screen, manifests to us indirectly its location at that instant. From Heisenberg's analogy, it will also follow that an act of measurement generates a location for the particle, which was not in space in the earlier probability state.

Is Heisenberg, nevertheless, using Aristotle's potentiality as an adequate analogy to the state of probability? Aside from the resemblance between these two concepts, there exist also fundamental discrepancies.

Aristotle's discussion on potentiality and actuality always goes hand in hand with the form-matter dichotomy. For him, matter exists only as potential without formal cause, and formal cause is associated with the actuality of a being. He uses the example of a house, in which the art of building houses is the formal cause of the housebuilding. It is coming to be a house while it is being built. When it is a house, it is no longer becoming a house, no longer being built. In *Physics*, Aristotle defines the material cause as “that out of which a thing comes to be and which persists”, such as the bronze of the statue.[21] By the potential state of matter, Aristotle does not mean that it does not have a location, for the fact that the bronze is not yet a statue does not prevent us from determining it in space and time: We know the bronze is here and now, even if it is not being built. On the other hand, he defines formal cause as “the form or the archetype, i.e. the statement of the essence, and its genera.”[22] When illustrating the nature of the formal cause, Aristotle only enumerates instances that are non-local: the relation of 2 to 1 as the formal cause of the octave, and generally numbers.[23] For how absurd is it to think that we can locate a ratio or a number in space? The ratio of 2 to 1 does not inform us of a spatio-temporal picture of an octave, but this picture is instead provided by the material cause of the octave: The octave does not have a location unless it is played on a musical instrument. The 2 to 1 ratio, however, does determine an octave mathematically though not locally, by distinguishing it from the ratios which correspond to other intervals. Aristotelian formal cause is therefore not spatial: It is not the form or shape of an object, but a non-local mathematical principle which governs an inanimate object or event.

21. Aristotle, *Physics*, trans. R. P. Hardie and R. K. Gaye, (New York: Random House, 1941), 194b 23-26.

22. Ibid., 194b 27-28.

23. Ibid., 194b 28-29.

If my previous interpretation of Aristotle's matter and form is valid, then it is not absurd to claim that Heisenberg's assignment of potentiality and actuality to two quantum states should be reversed. Albeit Heisenberg's claim, let us test whether the opposite situation works: The probability state of the quantum system corresponds to the Aristotelian actuality, while the post-observational state corresponds to the potentiality. The first question one might ask is: In what sense is the undisturbed probability state of a quantum system more "actual" than its particle state?

Let us first see what makes the post-observational state nonactual. After the measurement of the system takes place, even if we agree with Heisenberg in considering the space-time certainty of the system as the essence of actuality, this system is actual only in a qualified sense. It is the principle of indeterminacy he proposes that prescribes a limit to the degree of the space-time determination of the particle, which states that any pair of complementary physical quantities of a particle cannot be simultaneously measured. Suppose we precisely determine the position of an electron and therefore have complete knowledge about its location at one instant. We must then sacrifice our knowledge of its momentum, which means we cannot predict the electron's future course. When we determine its momentum accurately by sacrificing our knowledge of its position, by preserving the information of the future of the quantum system, we lose certainty about its present. If we are completely certain about the position of the electron, we are uncertain about its momentum and vice versa. Between the two extremities of the spectrum of such quasi-actuality, we are only left with a blurry picture: Neither the position nor the momentum can be accurately measured.

What makes the probability state of a quantum system more analogous to the Aristotelian state of actuality than potentiality? In the state before observation, the quantum system is governed by the probability function, which Aristotle would consider as the formal cause of the quantum system of the electron. Just as the 2 to 1 ratio determines the octave, the probability wave precisely determines the probability of the electron's presence in every point of space. The probability is not a compromise: The information provided by the probability function is precise and exhaustive. It does not matter whether the probability function can predict the electron's position with certainty, as long as we deny the space-time picture as being essential to the nature of the electron. When we detect the electron with the fluorescent screen, however, we are essentially disturbing the system and as a result, a compromise must be made for the sake of its space-time determination. A mathematical account of this compromise is that our observation collapses the wave function into one point, which means that all information about other points in space is suppressed. Therefore, the change from probability state to the observable state is opposite of the change from potentiality to actuality. For Aristotle, the change from potentiality to actuality is a process of perfection, for nature always tends towards the best, while in observing the probability state we are degrading our knowledge of the electron, from accurate prediction of its possible location, to an unsatisfactory picture of its "existence" smudged by the principle of uncertainty.

V. The probability function: a formal principle connecting the probability state and empirical events

> *Heraclitus says that for those who are awake there is a single, common universe, whereas in sleep each person turns away into his own private universe.*[24]

Heisenberg's indeterminacy principle is a compromise made in order to save the space-time description of an elementary particle. This implies, nevertheless, that if we no longer regard space-time determination as the ultimate criterion for reality, there is no need for the indeterminacy principle. Aristotle tells us that formal cause, which is closer to nature and reality, is often a mathematical principle and non-spatial. In quantum mechanics, we also have a candidate for the formal cause: the probability function. If Aristotle's account is valid, then the probability function is no longer an interpretation but exists in its own right. The probability function would no longer be a human invention or merely serve as a tool for us to understand the world better, but it has always existed as the Logos of nature herself.

How can we understand the probability function as a formal cause which transcends space and time? It is an a priori mathematical principle which unifies manifold empirical events throughout time and space. With the probability function as a formal cause, the boundary between the probability state and the empirical world drawn by Heisenberg's principle of indeterminacy disappears.

The probability state and empirical events are not completely separated from each other, since the prediction of a probability function is not completely random and erratic. The probability function still informs us of something true about the observable events happening in

24. Pseudo-Plutarch, *De superstition* 166c: ὁ Ἡράκλειτός φησι τοῖς ἐγρηγορόσιν ἕνα καὶ κοινὸν κόσμον εἶναι τῶν δὲ κοιμωμένων ἕκαστον εἰς ἴδιον ἀποστρέφεσθαι.

time and space, provided that there are enough numbers of such identical events. The probability function seems less powerful and accurate when predicting an individual event, than when foretelling the general trend of a cluster of identical events. This connection between probability state and real events is not confined only to the quantum level. It can also be demonstrated in predicting probabilistic events in our ordinary life, such as throwing dice.

There are two characteristics of this relation between the probability and the real events it predicts. First, the knowledge given by the probability needs no testing. Since a die has six surfaces with the same size and shape, we already know, even before throwing it, that the chance of getting number 3 is 1/6 or 16.7%. This knowledge is always accurate, regardless of the outcome of throwing the die. If after we throw the die for the first time we get the number 2 instead of 3, the probability of getting number 3 is still 16.7%, which is unaltered by this one event, even though it appears to be 0%. Probability is always an apriori knowledge about our possible experience, while it can be but need not be confirmed by our real experience. If we think of time and space as the conditions which make our experience possible, then the truth about the probability 16.7% does not rely on time and space, even though this truth can only be experienced by us through space and time.

Second, the truth about probability manifests itself not in an individual happening, but in the manifold. There must be enough events in order for us to experience the accuracy of a probability prediction. The greater number of real events we gather, the more likely it is for the probability function to correspond to our experience. If we throw the die only once and get number 2, the probability of getting number 3 (P_3) seems to be 0%. If we throw it again and get number 3, then P_3 appears to be 50%. But if we throw the die for a thousand times, P_3 will be much closer to our prediction of 16.7%. It is not unconceivable that if we throw the die infinitely many times the probability will be infinitely approaching 16.7%, though not exactly in a linear way. The probability may oscillate around 16.7%, just as in statistical sciences all the data oscillate around the value of the standard deviation. Even if after many throws the probability deviates significantly from 16.7%, we would blame the die, not the law of probability itself, since we regard the probability of 16.7% as a priori true which need no empirical testing. Probability is not unlike a mathematical limit which prescribes a boundary for possible happenings in space and time, but the limit itself exists apart from space and time. The probability function is a mathematical principle which unites manifold possible events whether they happen in space and time or not. Therefore, we can say that the probability function itself is a non-local principle.

How does a mathematical principle function as a unity that transcends time and space? Let us first explore an example in pure mathematics, in which the function unifies mathematical objects that are not in time and space. Here is the function which defines the class of all positive odd integers: $k = 2n + 1 (n >= 0)$. This one function alone determines what members will be included in this class. There is no need for us to enumerate all the numbers in this class, which are infinite, in order to prove this function to be true, just as we do not need to throw the die infinitely many times to test the accuracy of the probability 16.7%. Instead, the function $k = 2n + 1$ foretells all possible members in the class of positive odd numbers by defining one of the formal properties these members share: The possible ks are all equal to some non-negative even integers plus 1. In the same way, the probability of throwing dice informs us of one common property of all our future actions of throwing the die: We have a chance of 16.7% to get number 3 for each throw. Despite the similarity between the two mathematical principles, they differ in degree of certainty. From the function $k = 2n + 1$, we know exactly what members we will find in the class of positive odd integers, such as 1,3,5,7,9 and so on. A probability function does not tell us about the members with certainty, but still with accuracy, as the values will oscillate around the predicted value. For example, instead of (1,3,5,7,9……), the members might be (1.1, 3.09, 5.001, 7.000008, 9.111……), which are not exactly odd integers but are close to the predicted results. The law of the manifold also holds true in this case. With 1.1 and 3.09 as the only two members, we see these two members significantly deviate from the class of odd numbers, and it can represent also, for instance, the class of positive numbers, as we lack information about other members. Yet, the more such members we have, the more the probability function corresponds to a trend which confirms the class of odd positive integers.

After seeing the example of a pure mathematical function, let us unravel the mystery of the probability wave function in quantum physics, where the situation becomes more complicated because space and time are involved. The manifold members governed by the probability wave function are no longer just pure mathematical concepts like number, but are physical concepts such as the position and the momentum of a particle, the determination of which involves space and time. We can also demonstrate in the science of quantum physics, however, that the wave function transcends space and time by uniting manifold events across space and time. In one quantum system, suppose we have made a fixed number of observations within a space of *x* and a time period of *y*. If we change the extension of space and the duration of time in which the observations happen by altering the value of *x* or *y*, the predicting power of the probability function remains the same. The probability function, in terms of predicting the effects of multiple observations, is indifferent to the magnitude of time and space. Here, dimensionality becomes an irrelevant, or even useless, concept.

A perfect example of such a quantum system is a source of the cathode ray, from which the electrons pass through two slits and exhibit double-slit interference. Suppose we let 30,000 electrons pass through the slits and hit a fluorescent screen with area *x* in time *y*. If we let the area be fixed and just change the value of *y*, what appears on the screen will still be an interference pattern that confirms the prediction of probability function. Let us set *y* equals one minute, which means on average 500 electrons hit the screen per second. Or suppose *y* equals one day, and we observe the positions of 29,999 electrons in the first hour and wait for another 22 hours and 59 minutes before we determine the position of the last electron. In both cases, we will observe the interference pattern, for as long as it is a certain number of events which are to be determined, the amount of time which separates any of the two events does not matter. It is the wave function which describes the whole system that guides these discrete events, the spatial distribution of the 30,000 electrons, as one unity. On the other hand, if we fix the time *y* and change the area of the fluorescent screen, the same 30,000 electrons will always produce the interference pattern on the screen. This is because the coverage of the probability function is not limited to a certain region. It informs us of the probability of detecting an electron at every single location in the system under our observation, regardless of the size of the system. Even a fluorescent screen with the size of a football field would not be too large for the probability function of an electron. With millions of electrons passing through the double slit, we can make the interference pattern appear on the field. The probability function does not have physical dimensions, yet it gives us valuable information on a group of events happening in time and space.

VI. The probability function and the many-one dichotomy

> *He [Heraclitus] says that the wise thing is a single thing—knowing the plan which steers all things through all things.*[25]

Based on our previous thought experiments, what could be a possible explanation of the power of one probability function to guide manifold discrete physical events scattered throughout time and space? One way to reconcile this many-one dichotomy, is to eliminate the "many" altogether. The manifold is only an appearance: The electrons are ONE, though the oneness of the electrons is not an object of our possible experience, i.e. not in space and time. These identical electrons can be understood by us to be ONE as one form, because they follow the rule of one probability function, one formal mathematical principle. They become distinguished from each other as particles, only when we try to determine them in space and time as separate events.

The particles seem to us as discrete and manifold, simply because we cannot detect these events except in the context of space and time, not because they are multiple in their own nature.

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25. Diogenes Laertius 9.1: εἶναι γὰρ ἓν τὸ σοφόν, ἐπίστασθαι γνώμην, ὁτέη ἐκυβέρνησε πάντα διὰ πάντων.

We are never able to experience the oneness of these electrons in space and time, for the space-time determination of probabilistic events cannot be continuous, and there are always spatial or temporal gaps between two events. We consider throwing dice as a probabilistic event, because we assume that we can only observe the moment when a die falls on the table. Since after we toss the die, it takes a certain amount of time for it to move in the air before it falls and gives us a particular die number, the observations that we make are separated from each other temporally. In contrast, throwing dice no longer involves probability once we can continuously observe the motion of the die. This is not an insurmountable challenge: Once we measure the position and the momentum of the die at each moment we can calculate with Newton's laws and predict with certainty which side of the die will face up. The difficulty involved in making continuous observation in Newtonian world or about ordinary probabilistic events is subjective: It arises from our own limitation.

The non-continuity of observations at the quantum level bears a fundamentally different nature. It is not our subjective limitation but nature herself that forces us to experience the truth about one probability function in many separate events. It is not simply an issue of making discrete observations in continuous space and time. For if so, the implication would be that once we have the techniques to make continuous observations of electrons without breaks, we can see them as one continuous being in time and space. A continuous observation never takes place at the quantum level, because nature does not allow it. We can do that for ordinary probabilistic events such as throwing dice, since we assume that the die is in space and in time even when we are not observing it. As soon as we go down to the quantum level, what "happens" between the two discrete events does not even allow a space-time description. We can say that the particle does not exist in space and time before we measure it, despite the existence of time and space. A more radical interpretation will be that the space and time themselves do not exist for possible experience between our observations, and therefore the particle itself cannot exist because even the necessary conditions for its presence, space and time, are lacking. In either situation, space and time are no longer the Kantian apriori intuitions which are continuous and infinite. Space and time only come to exist when we disturb the probability state of a quantum system and thus force it into discrete pictures of "realities." The nature of the quantum system appears to be many separate events in space and time, but it is one mathematical principle, one reality, one non-spatial and non-temporal form. If we eliminate space-time as that which shapes and defines the unity of one event, we can grasp the idea that all separate particles as well as all distinct events can be connected as a unity, not spatially, but through one probability function.

Let us further unpack this many-one dichotomy in quantum mechanics by examining an analogy in the macro world. Suppose there is an opaque tank filled with water, and we cannot observe whatever happens inside. This water tank represents the quantum system we intend to investigate and is yet left undisturbed. We can observe the system by poking holes on the bottom surface of the tank. Each time we poke a hole at a particular location, a water stream is created which represents an individual particle which is forced by us into space and time. A water stream has no location until we poke a hole and cannot even be identified as an individual, just as an electron is not yet a particle in space and time until we measure its position or other phenomenal qualities such as momentum, energy and polarization. There is no way in which we can remove the entire bottom surface of the tank to reveal what is inside, because our observation of a quantum system cannot be continuous. Removing the bottom surface is identical to simultaneously poking infinite numbers of holes on the surface, which we cannot perform because of the fragmentary nature of our act of observation. Still, we can poke a number of holes and see water streams flowing from all of them. We tend to think of each water stream as an individual being, only because the water streams are separated from each other by space and time. This is the reasoning by which we claim this electron is not the same as another electron, as our measurement informs us that they occupy different spaces at different times. If we disregard their disparities due to their space-time distribution, however, it is not absurd for us to claim that these water streams are one unity. For these streams flow from the same source in the tank, no matter how far away the holes from which they flow are separated. In comparison, in the double slit interference of the electrons, we can treat all the electrons emitted from the cathode ray as one quantum system, since they share the same source and are governed by the same probability function. That is the reason why they form an interference pattern as long as we do not disturb this system by installing detectors at the two slits. The one quantum system does not become differentiated as individual electrons until the cathode ray strikes the fluorescent screen and makes its appearance in space and time. There exists, however, a fundamental distinction between our water tank analogy and the double-slit interference experiment. The unity of the water streams is the one water pond in the tank, which is still physical, but the unity of particles is formal. The water tank analogy implies that different water streams are still physically connected to each other in the tank, though in a space we cannot see. According to Einstein, Podolsky and Roson's theory, the water in the tank will be the "hidden variable" that preserves the idea of "locality," which is not yet covered by the science of quantum mechanics. While the science of quantum mechanics is proved to be complete by experiments on Bell's inequality, EPR's postulate of the existence of some physical connection between individual particles is inadequate. In a quantum system, different elementary particles are not connected physically, but are instead united mathematically by the probability function. What keeps these electrons as a unity is the probability function which is beyond our possible experience, instead of anything spatial or temporal. The notions of space and time imply anthropomorphism: They are the qualities we firmly hold on to as a secure foundation of all our possible experiences of nature, but the oneness of the probability function transcends space and time. By coercing the probability function into space and time, we are breaking up its unity into manifold individual beings or events. Ironically, it is we ourselves who disrupt the unity of nature we seek.

To see that the probability function is the ultimate concept of unity, let us explore what happens to the notion of space in the strange phenomenon of quantum entanglement. The quantum entanglement makes its first appearance as a thought experiment in EPR's paper whose aim is to prove the incompleteness of quantum mechanics.[26] Their theory is that, in the system of two entangled photons, once we determine the position of the one, the position of the other is simultaneously determined, regardless of the spatial distance between the two particles. The problem about this thought experiment is that it violates the principle of locality, because under

26. A. Einstein, B. Podolsky, N. Roson, "Can Quantum Mechanical Description of Physical Reality be Considered Complete?", in *Physical Review* 47 (1935), 777-80.

the constraint of the theory of special relativity no signal can travel at a speed faster than light. Since it is impossible for the determination of the position of the one photon to affect the position of the other instantaneously, EPR claims that quantum mechanics is essentially incomplete.

This problem can nevertheless be circumvented if we do not assume that any information has traveled from one photon to the other in space. In other words, by measuring one photon we are also introducing the entangled photon into a certain position, not locally but through one probability function which is the formal cause of both photons. There is no space between them before the probability function is disturbed, and the two photons behave as one in the probability state. The space between them is created only as the result of our measurement of one photon, an act of coercing the quantum system into space and time. Space is only a derivative concept from the disturbance of the original probability function. Therefore, even though the two photons appear to be separated in space after we determine their location, they still behave as one.[27] The

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27. I understand the relation between the two entangled photons as analogous to the love between Tristan and Isolde. These two lovers are one, but it is the light which separates them as two in space, as Tristan exclaims: "The day has made you glow and shine splendid in honor's light but infinitely far removed, just like the sun itself!" (Act II, Scene II) The two photons are one in the probability state, but it is our observation which splits them into two in space and time. Tristan and Isolde exalt the beauty of the night, for they can only be united in the night, not in the daylight where everything is exposed to human eyes. Their love, manifested by the waves of music (Wellen sanfter Lüfte), is the formal cause which binds them together as one unity that transcends space and time. The probability wave function, similarly, is the formal principle which forges the two photons into one entity even prior to the generation of space and time. In her parting song *Liebestod* (Act III, Scene III), therefore, Isolde regards their death not as a separation, but instead praises it as their eternal reunion in a state of supreme bliss (Höchste Lust). A loss of their bodily existence restores the two separate lovers to one unity beyond space and time. When two entangled photons are destroyed in space and time, that is when we stop measuring it, they are brought back to the undisturbed, un-localized, all-knowing state of probability. Two photons become one in the probability state, just as the two names "Tristan" and "Isolde" shall no longer distinguish one from the other in their duet: "No more naming, no

evidence for this unity is that two photons share the same properties such as polarization, momentum and spin (though with opposite spin directions).

The idea of space expanding between two photons can be made more intelligible by an analogy in the macro world, though an imperfect one. Imagine there is a balloon without air, and on the skin of the balloon let us mark two black dots which are so close to each other, perhaps with the distance of one millimeter, that one is indistinguishable from the other. Then let a person start blowing air into the balloon. The more the balloon expands, the further separated in space the two dots become, say from one millimeter to one centimeter, and eventually they are observed as two distinct dots rather than one. To connect this thought experiment back to the quantum entanglement, we can imagine the two black dots on the skin of the balloon as two entangled photons. At first, there is only one photon and no space exists in the probability state, just as there is no air in the balloon. Once a person starts blowing the balloon and as the space between the two photons expands, the effect of this expansion splits one photon into two. Still, I remark that this is an imperfect illustration, because as an event in classical physics it could not adequately imitate the condition of nonlocality. On one hand, the one millimeter distance between the two black dots already make them two, though they seem to us as one dot due to their proximity. In the case of quantum entanglement, however, there should be only one photon in the beginning. There is only one wave function and the photon is not in space before we measure it, because there is no space at all. On the other hand, the process of blowing the balloon

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more parting, newborn knowledge, newborn ardors, ever endless, both one mind: hotly glowing breast, love's supreme delight!" (Act II, Scene II) should be instantaneous in order to satisfy the condition for non-locality, so that the two dots do not move away from each other but appear separately in space instantaneously. Otherwise, the balloon is just a hidden variable which fits into the idea of locality. We can amend our example of the balloon, if we imagine that which makes the balloon expands is not the person who blows air into it as an efficient cause, but the probability function of the quantum system of the entangled photons as a formal cause.

Let us go back to the phenomenon of quantum entanglement. Before we locate one photon in space, as there is no space between two photons, there really is only one photon! We can regard the two entangled photons as essentially one quantum system, insofar as they are described by one common wave function. It even appears to me that "entanglement" is not an adequate word to capture the nature of this phenomenon, because the word entanglement implies a physical interaction between two objects. In the case we are discussing, there is really only one photon and therefore its needs neither physical contact nor interaction. Once we disturb one photon by measuring its position, we are altering the entire quantum system such that space emerges from nothingness as a new property of the quantum system, and the space breaks the one photon into two identical, discrete units instantaneously. There is no need for any information to pass between them, as there is no mechanical interaction between the two, because information is released to them through the one wave function which is non-local. The probability function, since it does not determine the photon locally, is not an efficient cause but a formal cause. It plays a similar role to that of Leibniz's God in mediating the "interaction" between two objects. According to Leibniz, two elastic objects which collide with each other change their shapes, not because they force each other to contract through immediate contact. Instead, it is God Who allows both objects to contract and fit into each other's shape in a non-local manner, as if they were compressing each other in space. Not unlike God, the probability wave function is the One that simultaneously distributes information to the two photons we experience in our space-time dimension.

Again, there really is one photon, but manifests itself as two in space and time. An act of measurement generates the position of the photon not only by creating it in some space which was already present, but also by creating the space as that which conditions the position of the photon. While we always experience the entangled system as two photons because our experience depends on the space-time dimension, we can understand the system as one, through comprehending the guiding power of probability waves.

VII. *h* as the primary substance

> *The totality of things, says Heraclitus, is an exchange for fire, and fire an exchange for all things, in the way goods are an exchange for gold, and gold for goods.*[28]

Once we grasp the probability function, how do we turn back and understand *h* in relation to it? Based on my previous arguments, I would treat the probability function not merely as an interpretation of the Schrodinger's wave equation. Probability is not merely a consequence of our language and knowledge: It exist apart from our process of knowing as a formal cause.

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28. Plutarch, *De E apud Delphous* 338d-e: πυρός τε ἀνταμοιϐὴ τὰ πάντα καὶ πῦρ ἁπάντων ὅκωσπερ χρυσοῦ χρήματα καὶ χρημάτων χρυσός.

One might ask: Of what substance is the probability function the form? The substance is h. The probability function is the formal cause, and h is the material cause, though h cannot be simply understood as the material cause in Aristotelian sense. While the material cause in Aristotle's *Physics* is visible, (the building blocks of a house, which we can experience in space and time, is the material cause of the house), the existence of h does not rely on space and time. Neither is h identical to the concept of matter in theories of classical physics. While in classical physics the quantity of matter, i.e. mass, is considered as the material cause and the primary substance, the substance h does not evolve to be mass until we measure it in time and space. h is a non-local substance which gives birth to the secondary physical concepts such as mass, energy, momentum and position once we coerce it into time and space through our act of measurement.

How can the h be understood as an immaterial substance? This thought seems absurd at first glance, but it does not come out of nowhere. Leibniz postulates a non-local substance which constitutes our universe. He names it "moving action". There is a striking similarity between Planck's constant and Leibniz's moving action: They have the same dimensions. Let us see why. One way to translate the dimensions of h into the product of two physical quantities is to multiply distance (*d*) by momentum (*p*). Since momentum can be further expressed as the product of mass (*m*) and velocity (*v*), $p = mv$, we can write the dimensions of h as $m \cdot v \cdot d$.

On the other hand, the calculation for motive action also involves the product of two physical quantities according to Leibniz. He calls one the "formal effect" and the other the "rigor of the force".[29] The formal effect (*f*) measures the effect produced by moving a massive object along a certain distance, and therefore has the same dimensions as Newton's turning power, the product of mass and distance: $f = m \cdot d$. By "formal", Leibniz suggests that this effect produced by a massive object is calculated solely in relation to its spatial displacement. The formal effect need not be seen through time, and time measurement is not relevant to it. Whether a massive object moves the distance in one day, or if it moves it in two days, the amount of formal effect does not vary. Therefore, our determination of the formal effect is only related to space. The second physical quantity rigor (*r*) measures the how quickly the effect of moving an object is produced. The effect of rigor can only be measured in time, for according to Leibniz rigor should be calculated by dividing distance over time. Thus, rigor has the same dimensions as speed: $r = v$. If we multiply formal effect by rigor to express the dimensions of moving action (*A*), we will have $A = f \cdot r = \ (md) \cdot v = m \cdot v \cdot d$, which has the same dimensions as Planck's constant: $h = m \cdot v \cdot d$.

Although Leibniz defines the moving action in relation to space and time, (since the formal effect is spatial, and the rigor is temporal), moving action itself is a non-spatial and non-temporal substance. While we need to measure these two effects from space and time, the quantity of moving action itself is conserved regardless of space and time. In other words, the existence of the moving action depends neither on space-time nor on our possible experience, and we make it an object of possible experience by observing its twofold effect in space and time. Before we observe how a massive object displaces itself within a certain amount of time, the

29. G.W. Leibniz, "Essay on Dynamics", in *Mathematische Schriften*, trans. C. Burke (1992), Vol. 6, 215-231.

moving action already presides as a cause that produces these subsequent effects. Another way to illustrate the a priori existence of moving action is the law of conservation of moving action. According to Leibniz, for any moment, the amount of the moving action in the world is conserved. It is impossible for moving action to be either created or to be destroyed. If the quantity of moving action is always constant throughout space and time, then it is not impossible for it to exist prior to space and time.

Despite the fact that they share the same dimensions, there is one fundamental difference between Planck's constant and moving action, which is related to the issue of continuity. A physical object in Leibniz's world can employ moving action continuously in space and time. This is because these three quantities whose product equals the moving action—mass, speed and distance—are continuous quantities. There is no ultimate unit for mass, as Leibniz thinks matter can be subdivided infinitely. The change of speed in classical physics is a continuous process in time: A moving object cannot double its speed instantaneously. In a similar manner, the change of displacement in classical physics is a continuous process in space: When an object is displaced from point A to point B, it must traverse all the space in between before reaching B. Since mass, speed and distance have a continuous range of values, the moving action itself must also be continuous. This also suggests that there is no real unit for the moving action: It is not quantized, and we can always subdivide moving action into smaller quantities.

In contrast, h is a constant, and thus we can understand it as a definite quantity of moving action. The existence of h entails the discreteness of moving action in the context of quantum mechanics. h is the real unit of moving action: It can neither be subdivided into smaller quantities, nor be changed continuously. In the world of quantum mechanics, the change of moving action is discrete, which takes gaps.

VIII. The connection between *h* and the probability function

> *Things grasped together: things whole, things not whole; something being brought together, something being separated; something consonant, something dissonant. Out of all things comes one thing, and out of one thing all things.*[30]
>
> *—Heraclitus*

What is it that quantizes the moving action as a homogenous substance into discrete units of *h*? It is the probability function that does the trick, and therefore the probability function can be thought of as the formal cause of *h*. If we follow Aristotle's scheme, then the continuum of moving action is the material cause of *h*, while the probability function gives a form to the matter by dividing the moving action into discrete units with magnitude *h*. Each unit of moving action is, however, separated from other units not because they are separated in space and time, but because they are made discrete by the probability function. Discreteness is itself a formal property, though it can be manifested in space and time in terms of discrete bodies, and the form of *h* is its discreteness. Both the quantity *h* and the discreteness of moving action are determined a priori by the wave function in the probability state, and this determination is prior to time and space. The form of *h* exists prior to the birth of space and time, since it already exists in the probability function before the probability state is coerced into the post-observational state: *h* is the only constant in the probability function, and this appearance is purely mathematical, not

30. Aristotle, *De mundo* 5.396b20: συλλάψιες· ὅλα καὶ οὐχ ὅλα, συμφερόμενον διαφερόμενον, συνᾷδον διᾷδον καὶ ἐκ πάντων ἓν καὶ ἐξ ἑνὸς πάντα.

spatial and temporal. The probability function as the formal cause is shaping *h* into one real unit, even before space and time, like a mathematical unit from a homogeneous substance of moving action. Hence, in the quantum world, the moving action has replaced mass as the material cause, for mass is a spatial quantity. Mass only comes to be in time and space through our observation, but the moving action already exists in the probability state prior to our observation.

There is also a reductio to demonstrate that the probability function must be the formal cause of *h*. If the probability function does not exist (as in the state of certainty described in the realm of classical physics), *h* does not exist either. The greater certainty there is in space and time, the lesser value *h* will possess. In the Newtonian world, since we believe that every object is determinate in time and space, the value *h* approaches 0 infinitely because we postulate energy as a continuum. Since if there is no probability function, there is no *h*, we can prove that the probability function is the necessary condition for the existence of *h* as a unit of substance. One might object to this argument by reminding us that this relation should be reversed: If there is no *h*, there is no probability function. This is because the discovery of *h* comes first and then leads to postulating the probability function only as an interpretation. This temporal sequence seems to imply that the probability function is secondary to the quantity *h*. This fact, nonetheless, does not exclude the possibility that the probability state exists in nature and is not merely a theory or a tool to aid in describing nature. As Aristotle says in *Metaphysics*: "[I]t is our task to start from what is more intelligible to oneself and make what is by nature intelligible intelligible to oneself."[31] It might well be that the probability function is prior to *h* by nature, even if we discover *h* earlier which is more intelligible to us. If the wave function already determines the form of *h* in the probability state, then Heisenberg, who relates the probability state to potentiality, seems to have misinterpreted Aristotle. The truth is, the probability state is already the state of actuality. Just as Aristotle says the actuality of the form and that of matter is one actuality,[32] the probability function and the unit *h* exist as the one actuality, before our experience of its emanations and derivative qualities.

The probability function, however, is a formal cause in more than one respect. According to Aristotle, a body can have multiple formal causes corresponding to different degrees of actuality. A statue made of gold, for example, has different formal causes in different stages. After it becomes the statue, the shape or structure of the statue is its formal cause. When the statue is still in the state of potentiality as a mere piece of gold, the gold also has a form even if it is not yet a statue. The probability function can be understood similarly as two formal causes. On one hand, it is the formal cause of *h*, since it creates units of moving action by acting on one homogenous substance, by giving each unit a discrete existence in the probability state. On the other hand, the function is the formal cause of one quantum system, by mathematically grouping definite numbers of *h* units into one quantum system and governing these units as one entity in the probability state. If the quantum system is analogous to the statue of gold, then *h* is the form

31. Aristotle, *Aristotle in 23 Volumes*, Vols.17, 18, trans. Hugh Tredennick, (Cambridge, MA: Harvard University Press; London: William Heinemann Ltd. 1933, 1989).

32. Aristotle, *Physics*, trans. R. P. Hardie and R. K. Gaye, (New York: Random House, 1941), 202a 18-22.

of gold while one quantum system with multiple *h* units is the form of the statue. The case of the probability function, nevertheless, differs from that of the statue of gold in that while the formal causes of gold and statue are from different sources, the probability function acts as a duel formal cause of both *h* and the quantum system it determines. These two formal respects of the probability function demonstrate the interplay between mathematics and the dichotomy of one and many. The probability function goes in both directions: First it forms multiple units of *h* out of one substance of moving action, and then it forms one quantum system out of these many units. Both processes happen in the state of probability, before the system manifests itself as particles or waves in our macro world.

Since there is only one quantum system in the state of probability, it is not meaningful to talk about "particles" until we measure them in space and time. As the word particle already implies spatial dimensions (we do not know something is a particle until we locate it in space), the multiplicity of particles is only a phenomenon in the post-observational state. One quantum system, when being disturbed by our measurement, can reveal itself to us as one or many particles, each particle carrying a definite amount of *h* units. Once we think of these "particles" as merely a result of our determination, and that they are governed by one probability function, then it is not impossible for us to explain some absurd quantum phenomena. In the case of the quantum entanglement of two photons, there is only one photon in the probability state governed by one probability function. As soon as we force the quantum system into our world through an act of measurement, it becomes two photons and space emerges between them just like an emanation from a light source. That is why both photons acquire the same position, momentum, polarization and spin instantaneously even if they are separated from each other in space.

By disturbing the function, we are materializing the quantum system composed of individual h units and forcing it into the realm of our possible experience. Before we measure the particle, the quantum system is not in space and time. Or we can say that space and time do not even exist before a particle is measured, just as there are no light rays before the source of the light comes into being. h exists already in the one quantum system governed by the one probability function, even if it does not yet make its appearance in our world.

How can we imagine that space and time do not exist before we experience a particle? Many physical quantities or concepts which describe the state of a particle in our world, including space and time, are derivatives of h as the result of our measurement. If we look again at the dimensions of h, we can see that h is binary: It can be the product of two other physical quantities with different pairs of combination. As discussed earlier, h has the dimensions of moving action in Leibniz' theory of dynamics: It is the product of formal effect (torque) and vigor (speed). We may notice another combination from Einstein, who discovers that h equals energy divided by frequency. As frequency is inversely proportional to period, h can also be thought of as the product of energy and time. The inequality ($\Delta x \cdot \Delta p \geq h$) which expresses Heisenberg's indeterminacy principle informs us that the dimensions of h equals those of the product of position (which involves distance) and momentum.

Whenever we measure or we experience the quantum system as a particle, we are dividing the system composed of unit h into two other phenomenal physical quantities. It is amazing how all the pairs of physical quantities, which I have mentioned as the binary emanations of *h*, are complementary, which means they are all subject to indeterminacy relations. Hence, it is the act of measurement that turns certainty into uncertainty: *h*, which is determinate a priori due to the probability function, becomes indeterminate in our world as two complementary physical quantities. Among those physical quantities, space (distance) and time are also in it. Consequently, it is not impossible for us to say that space and time do not come into being until we measure the quantum system, change the function, and split *h* into two phenomenal physical quantities. By altering the function, we change the formal cause, thus deforming *h* into different combinations of physical quantities in space and time. The probability function governs *h* as a true unity, but *h* splits into two complementary and blurry physical quantities as soon as we force the quantum system into our own world of "certainty". The transition from the probability state to the observed state is a process of degradation: This is a change from actuality into potentiality, from perfection to imperfection, from certainty to uncertainty, opposite to what Heisenberg claims about potentiality in *Physics and Philosophy*. *h* is more perfect than its derivative physical quantities such as position, momentum, space and time, in the way that a source is more perfect than its offspring. We are only able to "see" *h* as two complementary physical quantities, but *h* already exists in the quantum system as a unity defined by the probability function.

To summarize, the probability function is a formal cause in dual respects. First, it is the formal cause of *h* by determining this indivisible unit from the continuum of Leibniz's moving action. Second, it forges the multiple *h* units into one quantum system. With the presence of these two formal causes, the state of potentiality already attains it's a priori actuality. The particles we observe in our world are only imperfect manifestations of the probability function, just as complementary physical quantities with which we experience the quantum system are derivatives of *h* as the primary source. The probability function is no longer a compromise we make in the quantum world. Instead, it is a mode of perfection beyond our possible experience.[33]

Concluding remarks on probability, mathematics and nature

> *Nature loves to hide.*[34]
>
> —*Heraclitus*

Nature gives us whatever we wish for, and tricks us into believing that everything we take from her is real. It is an illusion that we can unveil nature through seeing her in space and time. The science of quantum mechanics warns us about our obsession with images, and tells us that we are missing the mark. Nature herself cannot be seen, and the only way to approach her is through mathematics, which cannot be seen but can only be understood. We see nature as fragmentary and manifold, but we can understand nature as a unity through the probability

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33. The perfection of the probability state resembles the perfection of God. If we compare the wave-particle duality in Quantum Mechanics to the duality of God and Man in Christianity, the collapse of a wave function, i.e., our space-time determination of a particle, is not unlike the incarnation of Jesus. Just as Jesus descends from the realm of the Father to our world by taking a bodily form perceivable in space and time, the particles come from the probability state to the Newtonian world by acquiring physical qualities such as momentum and position. This reminds me of the beginning of the Book of John: God was the Word (the Greek for "Word" is Logos, which could be interpreted as mathematics and intellect), and the Word became flesh (Jesus, His corporeal presence, space-time determination). Our act of measurement triggers the space-time occurrence of the particles by disturbing the probability function, while the original sin of human beings has infuriated God Who sent Jesus to be incarnated in our world. In other words, seeing is a sin: This is the reason why Adam and Eve need to cover their naked bodies with clothes after the Fall. When we stop measuring the particles and stop interfering with the perfect state of the probability function, or more generally if we finally decide to understand the world of quantum mechanics not with a space-time picture but through the probability function, then the quantum system is in a respect restored to its original nature. Similarly, the death of Jesus, the loss of His existence in space and time, is the only way to save human beings from their degraded state and restore them to the state of perfection.

34. Proclus, *Commentaire de la République II*: φύσις κρύπτεσθαι φιλεῖ.

function. The probability function is no longer just a descriptive tool to aid our search for nature, just as *h* is no longer a lucky guess in Planck's equation for the black body radiation. Mathematics should no longer be considered anthropomorphic: It is not just the precise language we have crafted in order to render a better account of nature. Mathematics is nature herself, just as the Pythagoreans believe that small number ratios constitute the musical cosmos. The probability function has thus prevailed over space and time and has established itself as the unifying principle of nature and the ultimate criterion for reality.