Euclid's Elements

all thirteen books complete in one volume

The Thomas L. Heath Translation
Dana Densmore, Editor

Green Lion Press
Santa Fe, New Mexico

Euclid's *Elements*

Book I

Definitions

1. A *point* is that which has no part.
2. A *line* is breadthless length.
3. The extremities of a line are points.
4. A *straight line* is a line which lies evenly with the points on itself.
5. A *surface* is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A *plane surface* is a surface which lies evenly with the straight lines on itself.
8. A *plane angle* is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called *rectilineal.*
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is *right,* and the straight line standing on the other is called a *perpendicular* to that on which it stands.
11. An *obtuse angle* is an angle greater than a right angle.
12. An *acute angle* is an angle less than a right angle.
13. A *boundary* is that which is an extremity of anything.
14. A *figure* is that which is contained by any boundary or boundaries.
15. A *circle* is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;
16. And the point is called the *centre* of the circle.
17. A *diameter* of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

Euclid's definitions, postulates, and common notions—if Euclid is indeed their author—were not numbered, separated, or italicized until translators began to introduce that practice. The Greek text, however, as far back as the 1533 first printed edition, presented the definitions in a running narrative, more as a preface discussing how the terms would be used than as an axiomatic foundation for the propositions to come. We follow Heath's formatting here. —Ed.

18. A *semicircle* is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.
19. *Rectilineal figures* are those which are contained by straight lines, *trilateral* figures being those contained by three, *quadrilateral* those contained by four, and *multilateral* those contained by more than four straight lines.
20. Of trilateral figures, an *equilateral triangle* is that which has its three sides equal, an *isosceles triangle* that which has two of its sides alone equal, and a *scalene triangle* that which has its three sides unequal.
21. Further, of trilateral figures, a *right-angled triangle* is that which has a right angle, an *obtuse-angled triangle* that which has an obtuse angle, and an *acute-angled triangle* that which has its three angles acute.
22. Of quadrilateral figures, a *square* is that which is both equilateral and right-angled; an *oblong* that which is right-angled but not equilateral; a *rhombus* that which is equilateral but not right-angled; and a *rhomboid* that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called *trapezia*.
23. *Parallel* straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Postulates

Let the following be postulated:

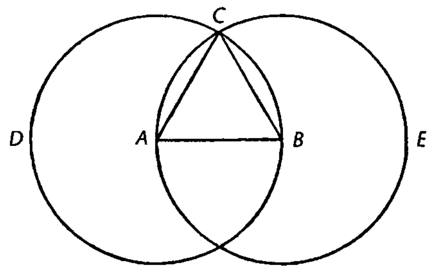
1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Common Notions

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Proposition 1

On a given finite straight line to construct an equilateral triangle.



Let AB be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line AB.
With centre A and distance AB let the circle BCD be described; [Post. 3]
again, with centre B and distance BA let the circle ACE be described; [Post. 3]
and from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined. [Post. 1]

Now, since the point A is the centre of the circle CDB,
AC is equal to AB. [Def. 15]

Again, since the point B is the centre of the circle CAE,
BC is equal to BA. [Def. 15]

But CA was also proved equal to AB;
therefore each of the straight lines CA, CB is equal to AB.

And things which are equal to the same thing are also equal to one another; [C.N. 1]
therefore CA is also equal to CB.

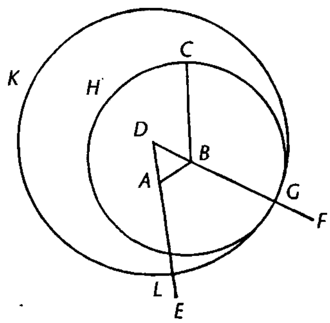
Therefore the three straight lines CA, AB, BC are equal to one another.

Therefore the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.

Being what it was required to do.

Proposition 2

To place at a given point [as an extremity][1] a straight line equal to a given straight line.

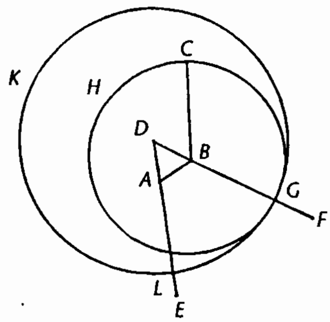


Let A be the given point, and BC the given straight line.

Thus it is required to place at the point A [as an extremity] a straight line equal to the given straight line BC.

From the point A to the point B let the straight line AB be joined; [Post. 1]
and on it let the equilateral triangle DAB be constructed. [I. 1]

1. Square brackets indicate material which Heath identified as having been supplied by him, adding clarification but not literally present in the Greek text. —Ed.



Let the straight lines *AE, BF* be produced in a straight line with *DA, DB*; **[Post. 2]**
with centre *B* and distance *BC* let the circle *CGH* be described; **[Post. 3]**
and again, with centre *D* and distance *DG* let the circle *GKL* be described. **[Post. 3]**

Then, since the point *B* is the centre of the circle *CGH*,

BC is equal to *BG*.

Again, since the point *D* is the centre of the circle *GKL*,

DL is equal to *DG*.

And in these *DA* is equal to *DB*;

therefore the remainder *AL* is equal to the remainder *BG*. **[C.N. 3]**

But *BC* was also proved equal to *BG*;

therefore each of the straight lines *AL, BC* is equal to *BG*.

And things which are equal to the same thing are also equal to one another; **[C.N. 1]**

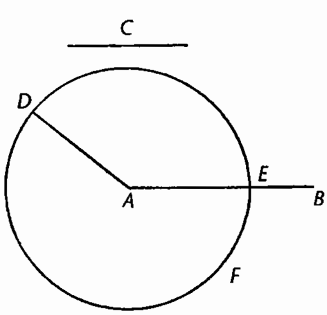
therefore *AL* is also equal to *BC*.

Therefore at the given point *A* the straight line *AL* is placed equal to the given straight line *BC*.

Being what it was required to do.

Proposition 3

Given two unequal straight lines, to cut off from the greater a straight line equal to the less.



Let *AB, C* be the two given unequal straight lines, and let *AB* be the greater of them.

Thus it is required to cut off from *AB* the greater a straight line equal to *C* the less.

At the point *A* let *AD* be placed equal to the straight line *C*; **[I. 2]**
and with centre *A* and distance *AD* let the circle *DEF* be described. **[Post. 3]**

Now, since the point *A* is the centre of the circle *DEF*,

AE is equal to *AD*. **[Def. 15]**

But *C* is also equal to *AD*.

Therefore each of the straight lines *AE, C* is equal to *AD*;

so that *AE* is also equal to *C*. **[C.N. 1]**

Therefore, given the two straight lines *AB, C*, from *AB* the greater *AE* has been cut off equal to *C* the less.

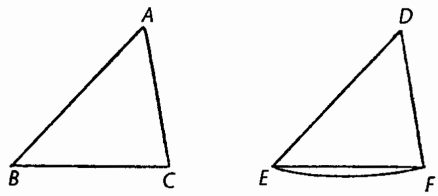
Being what it was required to do.

Proposition 4

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

Let *ABC, DEF* be two triangles having the two sides *AB, AC* equal to the two sides *DE, DF* respectively, namely *AB* to *DE* and *AC* to *DF*, and the angle *BAC* equal to the angle *EDF*.

I say that the base *BC* is also equal to the base *EF*, the triangle *ABC* will be equal to the triangle *DEF*, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend, that is, the angle *ABC* to the angle *DEF*, and the angle *ACB* to the angle *DFE*.



For, if the triangle *ABC* be applied to the triangle *DEF*, and if the point *A* be placed on the point *D* and the straight line *AB* on *DE*, then the point *B* will also coincide with *E*, because *AB* is equal to *DE*.

Again, *AB* coinciding with *DE*, the straight line *AC* will also coincide with *DF*, because the angle *BAC* is equal to the angle *EDF*;
hence the point C will also coincide with the point *F*, because *AC* is again equal to *DF*.

But *B* also coincided with *E*;
hence the base *BC* will coincide with the base *EF*,
and will be equal to it. [C.N. 4]

Thus the whole triangle *ABC* will coincide with the whole triangle *DEF*,
and will be equal to it. [C.N. 4]

And the remaining angles will also coincide with the remaining angles and will be equal to them,
the angle *ABC* to the angle *DEF*,
and the angle *ACB* to the angle *DFE*. [C.N. 4]

Therefore etc.

Q.E.D.[2]

Proposition 5

In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.

2. Q.E.D. stands for the Latin *quod erat demonstrandum,* that which was to have been demonstrated. The use of this and of Q.E.F., *quod erat faciendum,* that which was to have been done, is explained in the introduction. —Ed.